Ohio Operating Engineers

Entrance Exam Tune Up

Ohio Operating Engineers

Section One: Construction Math

Section Two: Mechanical Comprehension

OPERATING ENGINEERS
OHIO
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Local 18
APPRENTICESHIP & TRAINING

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Addition Problems

Addition is the term used to describe the combining two or more numbers together. For longer lists of numbers, it is easier to write the numbers in columns, and keep the columns in line with each other. Then you would perform your calculations at the bottom.

When we hear the terms 'ADD' or 'SUM', it means combine the numbers together. Sometimes our answers will be greater than 10, 100 or 1000, maybe even higher. We will need to understand how to "carry over" our values.

When we add 6 + 4, we get 10. We actually perform a simple carryover to get our answer. Both 6 and 4 are in the "ones" column, which is the first column to the **left** of the decimal. When we look at the number 10, the zero (0) is on the ones column, and the one is in the ones column!

$$\begin{array}{r} 6 \\ \underline{+4} \\ 10 \end{array}$$

That was a very simple explanation. Let's do another example. What is 36 + 9 = ?

Let's re-write the problem in columns, to see what should happen.

$$\begin{array}{r} 36 \\ \underline{+9} \\ ? \end{array}$$

We always begin working from right to left with our numbers. So we add 6 and 9 first. When we do, we get the result of 15. The 5 is in the ones column, and the 1 is in the tens column. We will write the 5 in the ones column of our answer, then "carry over" the 1 to the tens column. Write it **above** the tens column.

$$\begin{array}{r} 1 \\ 36 \\ \underline{+9} \\ 5 \end{array}$$

Now we need to ADD the one that we carried over, to the 3 that is already in the tens column. We get 4 as a result.

$$\begin{array}{r} 36 \\ \underline{+9} \\ 45 \end{array}$$

So now we see that 36 + 9 = 45.

Take a few minutes to work the problems below.

1.	4 + 4 =______	2.	3 + 3 =______	3.	1 + 4 =______
4.	1 + 3 =______	5.	4 + 3 =______	6.	2 + 4 =______
7.	1 + 2 =______	8.	3 + 2 =______	9.	1 + 1 =______
10.	3 + 1 =______	11.	5 + 6 =______	12.	4 + 4 =______
13.	1 + 5 =______	14.	1 + 4 =______	15.	6 + 5 =______
16.	3 + 6 =______	17.	1 + 3=______	18.	4 + 3 =______
19.	2 + 1 =______	20.	4 + 2 =______	21.	4 + 1 =______
22.	5 + 1 =______	23.	4 + 5 =______	24.	5 + 5 =______
25.	6 + 2 =______	26.	8 + 10 =______	27.	7 + 6 =______
28.	1 + 8 =______	29.	1 + 7 =______	30.	9 + 8 =______
31.	5 + 9 =______	32.	7 + 4 =______	33.	7 + 3 =______
34.	8 + 2 =______	35.	7 + 8 =______	36.	8 + 9 =______
37.	10 + 4 =______	38.	10 + 6 =______	39.	2 + 6 =______
40.	8 + 6 =______	41.	4 + 10 =______	42.	1 + 10 =______
43.	21 + 23 =______	44.	3 + 20 =______	45.	3 + 16 =______
46.	12 + 22 =______	47.	3 + 12 =______	48.	16 + 11 =______
49.	15 + 4 =______	50.	20 + 5 =______	51.	5 + 14 =______
52.	21 + 15 =______	53.	10 + 24 =______	54.	2 + 25 =______
55.	16 + 21 =______	56.	8 + 11 =______	57.	3 + 4 =______

Entrance Exam Tune Up

58. 23 + 14 =______
59. 8 + 21 =______
60. 15 + 24 =______
61. 22 + 14 =______
62. 21 + 23 =______
63. 17 + 16 =______
64. 3 + 20 =______
65. 22 + 19 =______
66. 12 + 22 =______
67. 17 + 8 =______
68. 17 + 19 =______
69. 19 + 21 =______
70. 24 + 9 =______
71. 25 + 16______
72. 2 + 25 =______
73. 49 + 55 =______
74. 42 + 37 =______
75. 8 + 47 =______
76. 9 + 39 =______
77. 51 + 45 =______
78. 28 + 53 =______
79. 9 + 28 =______
80. 39 + 26 =______
81. 16 + 6 =______
82. 40 + 20 =______
83. 36 + 11 =______
84. 47 + 13 =______
85. 42 + 46 = ______
86. 44 + 51 =______
87. 56 + 23 =______
88. 60 + 38 =______
89. 12 + 33 =______
90. 49 + 35 =______
91. 23 + 56 =______
92. 7 + 60 =______
93. 122 + 137 =______
94. 103 + 92 =______
95. 116 + 17 =______
96. 97 + 20 =______
97. 127 + 111 =______
98. 131 + 70 =______
99. 69 + 21 =______
100. 97 + 63 =______
101. 38 + 14=______
102. 98 + 48 =______
103. 88 + 26 =______
104. 115 + 30 =______
105. 113 + 103 =______
106. 125 + 110 =______
107. 140 + 56 =______
108. 758 + 65 =______
109. 623 + 86 =______
110. 429 + 91 =______
111. 209 + 42 =______
112. 565 + 124 =______
113. 753 + 154 =______
114. 632 + 281 =______
115. 824 + 716 =______
116. 738 + 670 =______
117. 907 + 798 =______
118. 669 + 588 =______
119. 836 + 721 =______
120. 928 + 334 =______

For fun:

1. 543 + 432 + 321 + 123 __________
2. 1,357 + 2,468 + 462 + 47 __________
3. 1,289,367 + 375,154 + 32,321 + 357 +5 = _________________
4. 333,333,333 + 477,777 + 88,888 + 55,515 + 54,321 + 903 =_____________________
5. (543 + 432 + 321 + 123 + 1,357 + 2,468 + 462 + 47 + 1,289,367 + 375,154) x .1 ________________

Subtraction

In math, to subtract means to take away from a group or number of items. When we subtract, the number of the items we are subtracting from becomes less. Subtraction is the opposite of addition. For simple subtraction problems, working side to side is not difficult. When we have multiple digits, it becomes easier to do subtraction when we write the problems in a column, and perform the calculations at the bottom. It is very important when re-writing the problem vertically, that the numbers are placed in exactly the right position. (ones, tens, hundreds, thousands,…..)

Sometimes it helps us to figure out subtraction problems, by remembering our addition problems. Many times, though, we need to think about subtraction a little differently than we thought about addition. Have you ever heard about backwards addition? Instead of 12-7 equals what? We can say "What do I add to 7 to get 12?"

When we are being asked to perform a subtraction problem, we will see or hear the terms "find the difference", "take away from", and "how many are left?"

When we get numbers that have more than one digit in them, the problems get a little more complicated, and we need to learn and understand a few rules.

Borrowing

Sometimes the number in the top row is smaller than the number in the bottom row. We will need to "borrow" from the next column to make our number big enough to subtract from.

Borrowing is a two-step process.

1. Subtract 1 from the top number in the column directly to the left.
   a. Cross out the number you're borrowing from, subtract 1, and write the answer above the number you crossed out.
2. Add 10 to the top number in the column you were working in.
   a. For example, suppose you want to subtract 386 – 94. The first step is to subtract 4 from 6 in the ones column, which gives you 2:

$$\begin{array}{r} 38\mathbf{6} \\ -\ 9\mathbf{4} \\ \hline \mathbf{2} \end{array}$$

When you move left to the tens column, however, you find that you need to subtract 9 *from* 8. But because 8 is smaller than 9, you need to "borrow" from the hundreds column. First, cross out the 3 and replace it with a 2, because 3 – 1 = 2:

$$\begin{array}{r} \overset{\mathbf{2}}{\cancel{3}}86 \\ -\ 94 \\ \hline 2 \end{array}$$

Next, place a 1 in front of the 8, changing it to an 18, because 8 + 10 = 18:

$$\begin{array}{rrr} 2 & & \\ \cancel{3} & \mathbf{18} & 6 \\ - & 9 & 4 \\ \hline & & 2 \end{array}$$

Now you can subtract in the tens column: 18 – 9 = 9:

$$\begin{array}{rrr} 2 & \mathbf{18} & 6 \\ - & \mathbf{9} & 4 \\ \hline & \mathbf{9} & 2 \end{array}$$

The final step is simple: 2 – 0 = 2:

$$\begin{array}{rrr} \mathbf{2} & 18 & 6 \\ - & 9 & 4 \\ \hline \mathbf{2} & 9 & 2 \end{array}$$

Therefore, 386 – 94 = 292.

In some cases, the column directly to the left may not have anything to lend. Suppose, for instance, you want to subtract 1,002 – 398. Beginning in the ones column, you find that you need to subtract 2 – 8. Because 2 is smaller than 8, you need to borrow from the next column to the left. But the digit in the tens column is a 0, so you can't borrow from there because the cupboard is bare, so to speak:

$$\begin{array}{r} 1{,}0\mathbf{02} \\ -\ 39\mathbf{8} \\ \hline \end{array}$$

When borrowing from the next column isn't an option, you need to borrow from the nearest "non-zero" column to the left.

In this example, the column you need to borrow from is the thousands column. First, cross out the 1 and replace it with a 0. Then place a 1 in front of the 0 in the hundreds column:

$$\begin{array}{rrrr} \mathbf{0} & & & \\ \cancel{1} & \mathbf{1}0 & 0 & 2 \\ & -\ 3 & 9 & 8 \\ \hline \end{array}$$

Now, cross out the 10 and replace it with a 9. Place a 1 in front of the 0 in the tens column:

$$\begin{array}{rrrr} 0 & \mathbf{9} & & \\ \cancel{1} & \cancel{10} & \mathbf{1}0 & 2 \\ & -\ 3 & 9 & 8 \\ \hline \end{array}$$

Finally, cross out the 10 in the tens column and replace it with a 9. Then place a 1 in front of the 2:

$$\begin{array}{rrrr} 0 & 9 & \mathbf{9} & \\ \cancel{1} & \cancel{10} & \cancel{\mathbf{10}} & \mathbf{1}2 \\ & -3 & 9 & 8 \\ \cline{2-4} \end{array}$$

At last, you can begin subtracting in the ones column: 12 – 8 = 4:

$$\begin{array}{rrrr} 0 & 9 & 9 & \\ \cancel{1} & \cancel{10} & \cancel{10} & \mathbf{12} \\ & -3 & 9 & \mathbf{8} \\ \cline{2-4} & & & \mathbf{4} \end{array}$$

Then subtract in the tens column: 9 – 9 = 0:

$$\begin{array}{rrrr} 0 & 9 & \mathbf{9} & \\ \cancel{1} & \cancel{10} & \cancel{10} & 12 \\ & -3 & \mathbf{9} & 8 \\ \cline{2-4} & & \mathbf{0} & 4 \end{array}$$

Then subtract in the hundreds column: 9 – 3 = 6:

$$\begin{array}{rrrr} 0 & \mathbf{9} & 9 & \\ \cancel{1} & \cancel{10} & \cancel{10} & 12 \\ & -\mathbf{3} & 9 & 8 \\ \cline{2-4} & \mathbf{6} & 0 & 4 \end{array}$$

Because nothing is left in the thousands column, you don't need to subtract anything else. Therefore, 1,002 – 398 = 604.

Let's so some subtraction problems.

1.	4 – 3 = ______	2.	5 – 4 =______	3.	4 – 2 = ______
4.	2 – 1 =______	5.	3 – 1 =______	6.	4 – 1 =______
7.	4 – 4 =______	8.	5 – 2 =______	9.	5 – 3 =______
10.	2 – 2 =______	11.	6 – 5 =______	12.	7 – 6 =______
13.	6 - 3 =______	14.	5 – 1 =______	15.	7 – 2 =______
16.	6 – 6 =______	17.	8 – 3 =______	18.	8 – 5 =______
19.	7 – 5 =______	20.	1 – 1 =______	21.	5 – 5 =______
22.	7 – 4 =______	23.	8 – 8 =______	24.	9 – 8 =______
25.	7 – 3 =______	26.	6 – 2 =______	27.	8 – 2 =______
28.	10 – 4 =______	29.	10 – 6 = ______	30.	8 – 6 =______

Entrance Exam Tune Up

31. 9 – 6 =______
32. 9 – 5 =______
33. 17 – 16 =______
34. 16 – 11 = ______
35. 15 – 4 =______
36. 21 – 11 =______
37. 18 – 16 =______
38. 24 – 12 =______
39. 18 – 11 =______
40. 18 – 12 =______
41. 15 – 11 =______
42. 24 – 14 =______
43. 19 – 17 =______
44. 17 – 4 =______
45. 22 – 12 =______
46. 19 – 8 =______
47. 25 – 15 =______
48. 17 – 10 =______
49. 22 – 19 =______
50. 17 – 8 =______
51. 20 – 5 =______
52. 24 – 9 =______
53. 25 – 16 =______
54. 21 – 15 =______
55. 23 – 14 =______
56. 22 – 14 =______
57. 25 – 8 =______
58. 18 – 12 =______
59. 39 – 26 =______
60. 16 – 6 =______
61. 40 – 20 =______
62. 36 – 11 =______
63. 47 – 13 =______
64. 56 – 23 =______
65. 49 – 35 =______
66. 54 – 34 =______
67. 49 – 27 =______
68. 43 – 30 =______
69. 14 – 12 =______
70. 35 – 14 =______
71. 28 – 16 =______
72. 58 – 34 =______
73. 46 – 40 =______
74. 26 – 22 =______
75. 42 – 11 =______
76. 45 – 21 =______
77. 47 – 36 =______
78. 42 – 37 =______
79. 51 – 45 =______
80. 56 - 23 =______
81. 60 – 38 =______
82. 52 – 33 =______
83. 43 – 39 =______
84. 60 – 21 =______
85. 57 – 28 =______
86. 43 – 27 =______
87. 103 – 92 =______
88. 127 – 111 =______
89. 97 – 63 =______
90. 38 – 14 =______
91. 98 – 48 =______
92. 88 – 26 =______
93. 115 – 30 =______
94. 140 – 56 =______
95. 149 – 93 =______
96. 122 – 86 =______
97. 133 – 84 =______
98. 129 – 81 =______
99. 41 – 23 =______
100. 123 – 66 =______
101. 108 – 96 =______
102. 149 – 50 =______
103. 57 – 35 =______
104. 142 – 68 =______
105. 106 – 66 =______
106. 107 – 73 =______
107. 209 – 42 =______
108. 229 – 104 =______
109. 565 – 124 =______
110. 669 – 588 =______
111. 836 – 721 =______
112. 624 – 385 =______
113. 632 – 281 =______
114. 753 – 154 =______
115. 928 – 334 =______
116. 991 – 598 =______
117. 799 – 549 =______

Multiplication

Multiplication is a method for us to do fast addition. It allows us to join groups, of equal sizes through repeated addition.

There are a few terms and symbols used when we need to use multiplication. All of them are interchangeable, and they mean the same thing. We will sometimes see the letter X, or an asterisk *, or a simple dot • . Each of these means that we should multiply.

When we see a multiplication problem such as:

$$4 \times 3 = ?$$

The problem is saying in essence: I have four (4) threes (3's), what is the total value? The problem would like this:

$$3 + 3 + 3 + 3 = ?$$

Then we can do the addition. 3 + 3 = 6, and 6 + 3 = 9, and then 9 + 3 = 12. The answer for 4 x 3 = 12.

So now we know that 4 X 3 = 12. What is the solution to the next problem?

$$3 \times 4 = ?$$

We now need three (3) fours (4's).

$$4 + 4 + 4 = ?$$

Then can do the addition. 4 + 4 = 8, and 8 + 4 = 12. So the answer to the problem 3 X 4 = 12.

Did you notice that the answer is the same? Even though the numbers were in a different order, the answer is exactly the same as before. That is because in multiplication, the order of the numbers does not matter.

$$4 \times 3 = 3 \times 4$$

"Tricks" with certain numbers.

Now there are some rules and tricks to know about multiplication. There will be many repeated patterns that you will be see.

When you see a ZERO (0), remember that ANYTHING multiplied by 0, will EQUAL 0.

When you see a ONE (1), ANYTHING multiplied by 1, will equal itself. 3 X 1 = 3

When you see a TWO (2), ANYTHING multiplied by 2 is DOUBLED. 4 X 2 = 8, 6 X 2 = 12. It is the same as ADDING the number 2 times, 4 + 4 = 8, or 6 + 6 = 12.

When you see a FIVE (5), Use what you know about counting by 5's. 3 X 5 = 15, (5 + 5 + 5 = 15).
4 X 5 = 20, (5 + 5 + 5 + 5 = 20)

When you see a NINE (9), You can use addition and subtraction. First subtract 1 from the number being multiplied by 9. Then determine what number you would need to make the number you just found equal to 9? Then you can write your two numbers together.

4 X 9 = ?

4 – 1 = $\underline{3}$

$\underline{3}$ + ? = 9

$\underline{3}$ + $\underline{6}$ = 9

36

So we can now see that 4 X 9 = 36.

When you see a TEN (10), Add a zero (0) to ANY number that you multiply by 10. 7 X 10 = 70.
11 X 10 = 110.

The same pattern can be followed for 100, 1,000, 10,000 or 1,000,000! Count the number of zeros you have, and add them to your number! 50 X 10,000 = ?. 50 has 1 zero of its own. 10,000 has four (4) zeros. Add the four (4) zeros onto the 50, then add your comma where needed. 50 X 10,000 = 500,000.

When you see ELEVEN (11), Think of 11 as two 1's. 4 X 11 = ? 4 X 1 = 4 and 4 X 1 = 4, so, 4 X 11 = 44.

Now that we have learned a few new tricks, we can perform some simple multiplication very easily.

The next thing that will help us with multiplication is some simple memorization.

The best thing to help us with memorization is repetition. Repetition means to repeat things over and over again, until they become easy to remember. We can do this with math, especially multiplication, because the answer is always going to be the same.

Multiplication Table

The multiplication table is a very simple method to help us figure out the answers to simple multiplication problems, and the it will also help us to memorize those answers, by reviewing the work we have done.

The tricks we learned about above, will also help us to solve the simple problems, and remember those answers.

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It is recommended that the first time you fill in the table, start with the upper left corner, in the first empty box, and work your way to the right, filling in each empty box as you go.

Each box represents a math problem. The numbers on the left label the ROW, and the numbers on the top label the COLUMN.

So in Row 1, Column 1. The math problem is 1 X 1 = ?. That is correct, the answer is 1. Now write that in the first empty box.

Row 1, Column 2. The math problem is 1 X 2 = ?. That is correct, the answer is 2. Write that in the next empty box.

Fill in the rest of your table. *HINT* Write your answers neatly! If you cannot read your answer, how will you know what it is or what it should be?

X	1	2	3	4	5	6	7	8	9	10	11
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											

Now see, that was pretty easy! Did you notice any patterns?

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Well, we get to do it again. Remember what we said about repetition? This time, as you fill in each square, say the math problem to yourself as you write in the answers. This time should be faster and easier than the last time. Remember, write your answers neatly!

X	1	2	3	4	5	6	7	8	9	10	11
1											
2											
3											
4											
5											
6											
7											
8											
9											
10											
11											

Did it seem even easier this time as compared to the first time? It should. And that is the idea. The more we practice something, the easier it becomes in accomplishing the task. We also begin to remember and see the patterns easier as well.

Now that we have some solid learning experience with multiplication, let's do some math problems!

1.	4 x 4 =____	2.	3 x 3 =____	3.	1 x 4 =____
4.	1 x 3 =____	5.	4 x 3 =____	6.	2 x 4 =____
7.	1 x 2 =____	8.	3 x 2 =____	9.	1 x 1 =____
10.	3 x 1 =____	11.	4 x 1 =____	12.	3 x 4 =____
13.	4 x 2 =____	14.	2 x 2 =____	15.	3 x 3 =____
16.	2 x 3 =____	17.	4 x 4 =____	18.	1 x 2 =____
19.	2 x 1 =____	20.	3 x 2 =____	21.	5 x 6 =____
22.	5 x 4 =____	23.	2 x 5 =____	24.	6 x 5 =____
25.	4 x 6 =____	26.	5 x 3 =____	27.	5 x 2 =____
28.	5 x 5 =____	29.	6 x 3 =____	30.	6 x 4 =____
31.	3 x 6 =____	32.	2 x 6 =____	33.	6 x 6 =____
34.	7 x 7 =____	35.	2 x 8 =____	36.	2 x 7 =____
37.	9 x 8 =____	38.	5 x 9 =____	39.	7 x 5 =____
40.	7 x 4 =____	41.	7 x 3 =____	42.	8 x 3 =____
43.	7 x 8 =____	44.	8 x 9 =____	45.	9 x 6 =____

Multiple Digit Multiplication

Up until now, we have focused on just single digit multiplication. Yes, our multiplication tables included the numbers 10 and 11. But, the pattern for answering those math problems was so simple, we did not need to think about the actual operation that was taking place. Now, we get to discuss and explain what needs to take place for successful multiplication of multiple digit numbers.

The process we need to follow in order to multiply multiple digit numbers is the same process as multiplying single digit numbers. We just need to make sure that we multiply EACH and EVERY digit.

For example:

10 X 2 = ?

We already know the “trick”. Just add a zero onto the 2. The answer is 20. But HOW is the answer 20?

Let's re-write the problem in columns, or vertically.

10

<u>X2</u>

?

What we need to do, in order to get the correct answer, is to multiply EACH digit. We always start on the RIGHT, and work our way through the problem to the LEFT. Just as we did when we were adding and subtracting numbers.

First we must multiply 2 x 0. And write the result.

10

<u>X2</u>

0

Remember ANY number multiplied by 0 equals 0. Now we must multiply the remaining digit by 2, and write the result.

10

<u>X2</u>

20

So now we can see WHY and HOW 10 x 2 = 20.

We follow the same process as we get larger numbers. Let's try one.

123

<u>X4</u>

?

Again, start on the right, and multiply 3 x 3.

123

<u>X3</u>

9

Then move on to the next digit. 3 x 2. And write the result.

123

X3

69

Finally, multiply the final digit. 3 x 1. And write the result.

123

X3

369

The answer is 369.

This process can be continued for any number of digits. What happens when we make the number that we are using to multiply a multi digit also? We follow the same pattern, with a slight adjustment in our process, so that we can keep all of our numbers straight and accurate.

For example.

11

X11

?

Just like in previous problems, start on the right, and work left. Multiply 1 x 1 and write the result.

1 1

X1 1

1

1 X 1 = 1. Then multiply the second digit. Write your result.

1 1

X1 1

1 1

We are finished with the first digit, now we must move onto the second digit. When we move over to new digits on the bottom, we need to use an indicator to let us know where we are in the process. We will use the numeral value of 0. Because we are working with the second digit, we will write our numbers on the second line. The first number we write is the 0, it is a placeholder. (Every time we move over to a new digit on the bottom, we add an additional placeholder).

1 1

X1 1

1 1

0

Now that our placeholder is entered, we perform the math, using the second digit. We will still work from right to left for our calculations. Multiply 1 x 1, and write the result starting next to our placeholder.

1 1

X1 1

1 1

1 0

Then we move onto the next digit. Multiply 1 x 1 and write the result.

1 1

X1 1

1 1

1 1 0

We have performed all of the calculations for each of our numbers. Now we ADD the results together.

1 1

X1 1

1 1

1 1 0

1 2 1

Now we can see that our result for the problem 11 x 11. The solution is 121.

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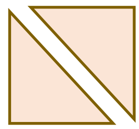
Now try some multiplication with multiple digits.

46.	9 X 10 =_____	47.	10 X 4 =_____	48.	10 X 7 =_____
49.	10 X 11 =_____	50.	11 X 9 =_____	51.	6 X 11 =_____
52.	10 X 3 =_____	53.	9 X 10 =_____	54.	9 X 11 =_____
55.	12 X 5 =_____	56.	12 X 8 =_____	57.	5 X 12 =_____
58.	13 X 15 =_____	59.	11 X 10 =_____	60.	13 X 12 =_____
61.	8 X 14 =_____	62.	11 X 7 =_____	63.	11 X 12 =_____
64.	15 X 7 =_____	65.	15 X 10 =_____	66.	14 X 11 =_____
67.	17 X 19 =_____	68.	15 X 14 =_____	69.	6 X 17 =_____
70.	18 X 16 =_____	71.	12 X 18 =_____	72.	15 X 11 =_____
73.	14 X 7 =_____	74.	17 X 8 =_____	75.	16 X 17 =_____
76.	20 X 14 =_____	77.	11 X 19 =_____	78.	21 X 17 =_____
79.	26 X 28 =_____	80.	23 X 21 =_____	81.	9 X 25 =_____
82.	27 X 24 =_____	83.	17 X 27 =_____	84.	22 X 16 =_____
85.	25 X 11 =_____	86.	23 X 24 =_____	87.	24 X 26 =_____
88.	29 X 15 =_____	89.	30 X 21 =_____	90.	26 X 20 =_____
91.	92 X 92 =_____	92.	71 X 63 =_____	93.	17 X 79 =_____
94.	18 X 67 =_____	95.	86 X 75 =_____	96.	50 X 88 =_____
97.	19 X 49 =_____	98.	67 X 45 =_____	99.	61 X 22 =_____
100.	78 X 25 =_____	101.	71 X 77 =_____	102.	75 X 85 =_____
103.	94 X 91 =_____	104.	100 X 64 =_____	105.	101 X 37 =_____
106.	105 X 99 =_____	107.	137 X 72 =_____	108.	159 X 104 =_____

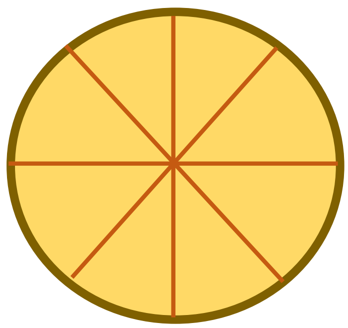
Division

Division is the opposite of multiplying. It is the process of breaking an object into an equal number of parts.

If we cut a sandwich into 2 halves, we have divided that sandwich.



If we cut a pizza into 8 equally sized pieces, we have divided that pizza.



If we make change for a dollar, we can make 2 equal values of 50¢, or 4 equal values of 25¢,

or 10 equal values of 10¢, or 20 equal values of 5¢, or 100 equal values of 1¢.

When people hear the term "Long division", they get intimidated. But there is no need to be concerned. We will discuss the steps needed to successfully do the math.

First, we need to know some of the terms used in division.

Dividend, Divisor, Quotient, and Remainder.

$$\underset{\text{Dividend}}{12} \div \underset{\text{Divisor}}{3} = \underset{\text{Quotient}}{4}$$

The **dividend** is the number being divided. The **divisor** is the number of parts we are dividing by. The **quotient** is the answer. Notice we did not label the remainder? That is because in this instance, there is not one! The **remainder** is what is left over, or remains behind when we are finished.

You probably recognize the symbol we used to let us know it was a division problem. " ÷ "

Another way we may be told that it is a division problem is the use of a forward slash or a backslash.

12/3 = ?   16\4 =?

Another symbol that may be used is the right parentheses, with a long arm attached to it.

It does not have a technical name. We can just call it the long division symbol. $\overline{)}$

When this symbol is used, the math problem is written in a specific way. The dividend goes inside, and the divisor goes outside. The problem below would be read as 12 divided by 3.

$$3\overline{)12}$$

What they want to know is, how many 3's fit equally into the number 12?

We can figure this out several ways. The first way is by simple addition.

3 + 3 = 6 (There's two 3's)

6 + 3 = 9 (Now there are 3)

9 + 3 = 12 (Now there are 4)

3 + 3 + 3 + 3 = 12

Counting by 3's : 3, 6, 9, 12.

Or we can REMEMBER our Multiplication chart!

$$3 \times 1 = 3 \ , \ \ 3 \times 2 = 6 \ , \ \ 3 \times 3 = 9 \ , \ \ 3 \times 4 = 12$$

The multiplication chart saves the day! Memorizing those answers is coming in handy.

But there is a little more to understand about division. When working out a division problem, we will use multiplication, and subtraction to complete the process. This is why we covered addition, subtraction and multiplication before we began discussing division. Let's look at an example.

$$121 \div 11 = ?$$

We need to re-write the problem vertically, using our long division symbol.

$$11 \overline{)\,121}$$

When we re-write our problem, the dividend (the number being divided) goes inside the symbol, and the divisor (the number we are dividing by) goes outside.

So, we want to know how many 11's fit equally into the number 121? In division, we work from left to right. In addition, subtraction and multiplication we worked from right to left. We must work with the entire number outside the symbol, and then begin working with the number inside the symbol, beginning on the left side.

How many 11's fit into the number 1? The answer is 0. 11 will not fit into 1. For this example, let's go ahead and write down that 11 will not fit into 1. We will write it on the line directly above the number 1.

$$\begin{array}{r@{\,}l} & 0 \\ 11 & \overline{)\,1\ 2\ 1} \end{array}$$

We need to move to the right including the next number. Now, how many 11's will fit into 12? Yes, that is correct, just 1. Let's write this down.

$$\begin{array}{r@{\,}l} & 0\ 1 \\ 11 & \overline{)\,1\ 2\ 1} \end{array}$$

Now, we need to use our multiplication. We are going to prove ourselves "right." We need to multiply the 1 times the *divisor (the number outside the symbol)*, and write it down.

```
      0 1
11  )1 2 1
      1 1
      ---
```

11 times 1 is 11, and we wrote it directly below the 12, the number we are working with. Now we must subtract 11 from 12. We get 1, and write it down.

```
      0 1
11  )1 2 1
      1 1
      ---
       0 1
```

At this moment we can check that we made the right choice, and determined that only one "11" could fit into 12. Is 1 less than 11? Yes, it is. When we do these types of problems, we always have a method to verify we made a proper choice. When we do our subtraction, and compare our result to the divisor, if the result is GREATER THAN the divisor, we need to pick a number large enough to make the result LESS THAN the divisor.

We still have more numbers inside the symbol, and we need to use them all to get a complete answer. So, the next step, we bring down the next number, and write it next to the number we just found.

```
      0 1
11  )1 2 1
      1 1
      ---
      0 1 1
```

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Now we see that we still have “11” inside the symbol. We are trying to make whatever is inside the symbol equal to 0. Now, how many 11’s fit into 11? And the answer is 1. Write this down. And do your multiplication. Write this down too. Finish with your subtraction, and write that down as well.

$$
\begin{array}{r}
0\,1\,1 \\
11\,\overline{)\,1\,2\,1} \\
\underline{1\,1} \\
0\,1\,1 \\
\underline{1\,1} \\
0
\end{array}
$$

So, the number that is written above the line, that is our result. It is also called the Quotient. It is just a fancy name for answer. Notice that when we subtracted our results from one another, the answer was “0”. That means there is no remainder. The answer to 121 ÷ 11 = 11.

It is very important to keep all of your number columns straight, so that you do not get lost in your steps. It also makes it easier to re-trace your steps if a mistake is made.

Let’s do some more division problems.

Division

1.	16 ÷ 4 =_____	2.	9 ÷ 3 =_____	3.	4 ÷ 1=_____
4.	3 ÷ 1 =_____	5.	12 ÷ 4 =_____	6.	8 ÷ 2 =_____
7.	2 ÷ 1 =_____	8.	6 ÷ 3 =_____	9.	1 ÷ 1 =_____
10.	3 ÷ 3 =_____	11.	4 ÷ 4 =_____	12.	12 ÷ 3 =_____
13.	8 ÷ 4 =_____	14.	4 ÷ 2 =_____	15.	9 ÷ 3 =_____
16.	30 ÷ 5 =_____	17.	16 ÷ 2 =_____	18.	5 ÷ 1 =_____
19.	4 ÷ 1 =_____	20.	30 ÷ 6 =_____	21.	18 ÷ 3 =_____
22.	2 ÷ 2 =_____	23.	20 ÷ 4 =_____	24.	25 ÷ 5 =_____
25.	12 ÷ 6 =_____	26.	24 ÷ 6 =_____	27.	20 ÷ 5 =_____

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28. 6 ÷ 2 =______	29. 80 ÷ 8 =______	30. 42 ÷ 7 =______
31. 72 ÷ 9 =______	32. 45 ÷ 5 =______	33. 28 ÷ 7 =______
34. 21 ÷ 7 =______	35. 16 ÷ 8 =______	36. 56 ÷ 7 =______
37. 72 ÷ 8 =______	38. 40 ÷ 10 =______	39. 60 ÷ 15 =______
40. 48 ÷ 8 =______	41. 40 ÷ 4 =______	42. 10 ÷ 2 =______
43. 182 ÷ 13 =______	44. 110 ÷ 11 =______	45. 24 ÷ 2 =______
46. 30 ÷ 3 =______	47. 156 ÷ 13 =______	48. 98 ÷ 7 =______
49. 21 ÷ 3 =______	50. 70 ÷ 10 =______	51. 27 ÷ 9 =______
52. 48 ÷ 12 =______	53. 132 ÷ 11 =______	54. 143 ÷ 11 =______
55. 84 ÷ 14 =______	56. 150 ÷ 15 =______	57. 27 ÷ 3 =______
58. 117 ÷ 13 =______	59. 84 ÷ 6 =_______	60. 30 ÷ 2 =______
61. 483 ÷ 21 =______	62. 288 ÷ 18 =______	63. 80 ÷ 4 =______
64. 68 ÷ 4 =______	65. 418 ÷ 22 =______	66. 264 ÷ 12 =______
67. 48 ÷ 4 =______	68. 187 ÷ 17 =______	69. 153 ÷ 17 =______
70. 75 ÷ 15 =______	71. 120 ÷ 20 =______	72. 342 ÷ 18 =______
73. 399 ÷ 19 =______	74. 240 ÷ 24 =______	75. 400 ÷ 25 =______
76. 315 ÷ 21 =______	77. 240 ÷ 10 =______	78. 75 ÷ 3 =______
79. 700 ÷ 25 =______	80. 440 ÷ 22 =______	81. 144 ÷ 6 =______
82. 147 ÷ 7 =______	83. 598 ÷ 26 =______	84. 432 ÷ 16 =______
85. 112 ÷ 7 =______	86. 294 ÷ 21 =______	87. 252 ÷ 21 =______
88. 133 ÷ 19 =______	89. 192 ÷ 24 =______	90. 528 ÷ 22 =______
91. 598 ÷ 23 =______	92. 377 ÷ 29 =______	93. 600 ÷ 30 =______
94. 144 ÷ 8 =______	95. 475 ÷ 25 =______	96. 377 ÷ 13 =______
97. 783 ÷ 29 =______	98. 288 ÷ 9 =______	99. 280 ÷ 10 =______
100. 792 ÷ 22 =______	101. 210 ÷ 10 =______	102. 560 ÷ 28 =______
103. 126 ÷ 14 =_______	104. 476 ÷ 28 =______	105. 928 ÷ 29 =______
107. 1258 ÷ 34 =______	108. 1085 ÷ 35 =______	109. 1085 ÷ 31 =______
110. 1080 ÷ 40 =______	111. 722 ÷ 19 =______	112. 5475 ÷ 73 =______
113. 1190 ÷ 35 =______	114. 1175 ÷ 25 =______	115. 5040 ÷ 70 =______

Decimals- Addition and Subtraction

Now that you have had practice with the four primary math equations; addition, subtraction, multiplication, and division. We are able to introduce decimals. In all our previous examples and exercises, our solutions worked out exactly, they were all WHOLE NUMBERS. Now we can introduce portions of numbers, or decimals.

When we discuss decimals, we need to understand a few concepts and terms.

When we write our numbers, we normally exclude the decimal when the number is EXACTLY what it should be. 1, 2, 3, 4 5, and so on. BUT, if the number is a little larger, or a little smaller, it means that it is not EXACTLY 1, or 2, or 3, and so on. So we now have a portion of a number.

1.01 Is a number, but it is not a whole number. It is not EXACTLY 1, it is MORE THAN 1, it is LESS THAN 2.

0.98 is a number, but it is not a whole number. It is MORE THAN 0, but LESS THAN 1.

These are portions of numbers. Any number that appears on the LEFT of the decimal point, IS a whole number. Any number that appears on the RIGHT side of the decimal, is a PORTION of a number.

The more numbers we have on the left of the decimal, the LARGER the number is. The more numbers we have on the right of the decimal, the SMALLER the number is.

Each “place” on both sides of the decimal, has a specific name to identify the VALUE of the number that is entered there. In the addition section we began discussing these names of values.

The first “place”, to the LEFT of the decimal, are the ONES.

1.0

The second “place” to the LEFT of the Decimal, are the TENS.

10.0

The third “place” to the LEFT of the decimal, are the HUNDREDS.

100.0

The fourth “place” to the LEFT of the decimal, are the THOUSANDS.

1000.0

What is the value of the fifth place? The sixth place? The ninth place?

10,000.0 100,000.0 100,000,000 .0

When we go the other direction, to the RIGHT of the decimal point, the places have names for their values too.

The first place to the RIGHT of the decimal, is named TENTHS.

0.1

The second place to the RIGHT of the decimal, is named HUNDREDTHS.

0.01

The third place to the RIGHT of the decimal, is named THOUSANDTHS.

0.001

The fourth place to the RIGHT of the decimal, is named TEN THOUSANDTHS.

0.0001

How would you say this number? 2,178.

Two Thousand, one hundred and seventy-eight.

How about this number? 2.17

Two and seventeen one-hundredths.

Notice we said two AND seventeen one-Hundredths. The word AND indicates we are recognizing the decimal point, AND including more numbers, or including a portion of a number. You might also say "Point" when speaking the number value, but do not say "dot" for the decimal point. We do live in the age of the internet and computers, and it may seem OK to say two "dot" seventeen one-hundredths, but it is incorrect, and you will be mis-understood.

Now that we have some new understanding, we can build on it.

This is not a perfect world. All the math problems do not work out perfectly, or exactly. How many times have you been at the grocery store and your items added up perfectly to a whole dollar amount? We know it does happen, but not very often.

We are sure that you have noticed that when buying groceries, everything has its own value, or cost.

Let's look at a simple grocery list.

1- Gallon of Milk	\$2.69
1- Loaf of Bread	\$1.49
1- Dozen Eggs	\$2.79
1- Pound of Bacon	\$4.99
1- Pound of Deli Sliced Turkey	\$5.29

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1- Pound of Colby Jack Cheese	$5.29
1- Large Bag of Chips	$4.49
1- 6-Pack of Pepsi/ Coke	$3.99
1- Family Pack (4) Boneless Strip Steak	$30.33
1- 5lb Bag of Idaho Potatoes	$2.49
2- 14.5 oz Cans Green Beans	$1.50
1- 12 ct Box Little Debbie Oatmeal Crème Pies	$2.59

How much will all this cost? Well, we need to add it all together. When adding or subtracting decimals, we MUST keep the numbers in line. The decimal number must "line-up" in every number. If the numbers are not lined up and kept in order, or if the decimal point is not lined up properly, the calculations will be incorrect.

How much was our grocery list? Did you get $67.93?

Let's do some addition and subtraction with decimals. *Hint: It may be helpful to re-write the problem vertically, in order to line up the decimals.

1. 8.0 + 9.0 =_____
2. 0.2 + 8.0 =_____
3. 0.8 + 7.0 =_____
4. 2.0 + 0.5 =_____
5. 3.0 + 0.2=_____
6. 6.0 + 0.2=_____
7. 7.0 + 7.0 =_____
8. 9.0 + 4.0 =_____
9. 3.0 + 6.0 =_____
10. 9.2 + 0.9 =_____
11. 0.8 + 0.9 =_____
12. 0.5 – 0.3 =_____
13. 0.7 – 0.2 =_____
14. 1.3 – 0.9 =_____
15. 0.7 – 0.4 =_____
16. 8.0 – 6.2 =_____
17. 9.2 – 2.7 =_____
18. 10.0 – 5.05 =_____
19. 8.10 + 9.10 =_____
20. 0.15 + 7.80 =_____
21. 0.85 + 7.40 =_____

22. 1.8 + 0.48 =_____ 23. 2.8 + 0.13 =_____ 24. 7.00 + 7.60 =_____

25. 0.55 + 0.61 =_____ 26. 1.80 – 0.48 =_____ 27. 2.80 – 0.13 =_____

28. 6.00 – 0.21 =_____ 29. 9.30 – 4.00 =_____ 30. 7.20 – 0.65 =_____

31. 7.20 – 4.60 =_____ 32. 6.81 – 0.91 =_____ 33. 5.80 – 0.76 =_____

34. 0.60 – 0.45 =_____ 35. 0.76 – 0.68 =_____ 36. 9.70 – 5.9 =_____

37. 8.50 – 0.95 =_____ 38. 0.810 + 0.910 =_____ 39. 0.015 + 0.780 =_____

40. .280 + .013 =_____ 41. 6.0 + 0.021 =_____ 42. 1.500 + .18 =_____

43. 1.500 – 0.180 =_____ 44. 7.2 - .0655 =_____ 45. .580 – 0.076 =_____

46. 6.800 – 0.910 =_____ 47. 0.320 - .170 =_____ 48. 7.542 – 6.753 =_____

49. Six Tenths =_____ 50. Ten Tenths =____

51. Twelve Hundredths =_____ 52. Three thousandths =_____

53. Three and forty-nine hundredths =_____

54. Five hundred ninety-four and two hundred thirty-six thousandths =_____

55. Seven thousand seventeen thousandths =_____

Decimals- Multiplication

Doing addition and subtraction with decimals is not difficult. We must simply remember to keep our columns properly lined up, and we can add or subtract as needed.

With multiplication of decimals, it is even easier.

We are not required to line up our decimal points! In fact, we can "ignore" the decimal point, until we are ready for our final answer. Let's demonstrate.

$$\begin{array}{r} 1.2 \\ \underline{\times\ 3} \\ ? \end{array}$$

We can first approach this problem exactly the same as we did with regular numbers. Multiply each of the numbers as required. Working right to left. (2 x 3, write it down. 1 x 3 and write it down).

$$\begin{array}{r} 1\ .\ 2 \\ \underline{\times\quad 3} \\ 3\quad 6 \end{array}$$

So far our answer is 36, but we still have to USE our decimal point(s).

We must simply count the number of decimal places in the original problem. In our example problem above, we only have 1 decimal place shown. So now we count from the right, the number of decimal places, and enter the decimal point there.

$$\begin{array}{r} 1\ .\ 2 \\ \underline{\times\quad 3} \\ 3\ .\ 6 \end{array}$$

The correct answer is 3.6.

Let's do another example with a few more decimals.

2.15

<u>X3.6</u>

?

The work should be as follows:

2 . 1 5

<u>X 3 . 6</u>

1 2 9 0 (6 x each #)

<u>6 4 5 0</u> (Enter the 0 for the placeholder, the 3 x each #)

7 7 4 0 (Add them together)

Now the only step we have left is to add in our decimal point. How many decimal places are in the original problem?

2 . 1 5

<u>X 3 . 6</u>

1 2 9 0

<u>6 4 5 0</u>

7.7 4 0

There were 3 decimal places in the original problem. The correct answer is 7.740.

*A helpful tip: Remember that 1.0 is the SAME as 1. Even with the answer for our example problem above. 7.74 is the SAME as 7.740.

IF you get a multiplication problem such as 3.0 x 4.0 = ? There are no decimals in the problem. All of the real values are on the LEFT of the decimal. You can omit the 0's from the problem and it becomes 3 x 4 = ?

The same is true for a problem such as 0.30 x 0.40 = ? The problem is actually .3 x .4 = ? There is no value on the LEFT of the decimal point. There are no values after the .3 or the .4. You can "remove" the 0's. But remember there are still 2 decimal points in the problem!

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Let's do some exercises.

1.	8 x 0.9 =____	2.	6 x 0.2 =____	3.	2 x 0.60 =____
4.	7 x 0.5 =____	5.	2 x 0.5 =____	6.	0.5 x 3 =____
7.	7 x 0.4 =____	8.	0.2 x 8 = ____	9.	7 x 0.7 =____
10.	0.8 x 9.0 =____	11.	0.9 x 6 =____	12.	6 x 0.8 =____
13.	81 x 9.1 =____	14.	62.0 x 1.5 =____	15.	17 x 6.6 =____
16.	74 x 4.9 =____	17.	18 x 4.8 =____	18.	4.4 x 28 =____
19.	66 x 3.5 =____	20.	2.1 x 77 =____	21.	56 x 8.1 =____
22.	4.1 x 93 =____	23.	99 x 6.3 =____	24.	18 x 8.9 =____
25.	35 x 8.5 =____	26.	95 x 8.6 =____	27.	74 x 2.7 =____
28.	0.99 x 63.00 =____	29.	5.60 x 8.10 =____	30.	0.35 x 45.00 =____

Dividing Decimals

We have learned that there are certain rules for adding and subtracting decimals. Keep the decimals lined up!

We have learned a simple rule for multiplying decimals. Multiply like normal, ignoring the decimals, until you are ready for the final answer. When you are ready, count the total number of decimal places in the original problem, and then add them to the answer.

Now we have two simple, but very important rules, for dividing with decimals.

Rules

Rule 1.

When we have a decimal value in the dividend, and only the dividend, we must make sure that the decimal stays properly aligned in the answer. Remember the dividend is the number "inside" our long division symbol. It is the number being divided BY another number. The divisor.

Rule 2.

If we have a decimal value in our divisor, we need to get rid of it. We do not want any decimals in our divisor. According to the almighty rules of math, we are not allowed to do the math with a decimal value in our divisor. Remember the divisor is the number "outside" our long division symbol. The divisor is the number being divided into another number.

Rule number 1 does not seem very difficult, because it is not. The easiest way to make sure you follow this rule is, before you start the problem, mark your decimal point ABOVE the long division symbol, directly above its location in the problem. This will allow us to keep it "in the right place." If our final answer does not include any numbers to the right of the decimal, we do not need to mention it. If our final answer does include numbers to the right of the decimal, it has been taken care of.

How do we get rid of the decimal, like we are told to do in Rule 2?

That's easy. We multiply.

What do we multiply by? We multiply by whatever we need to make the decimal "disappear."

We use powers of 10.

Remember when we talked about multiplication? When we multiply by 10, we "add a zero." If we multiply by 100, we add two zeros. 1,000 will mean we add 3 zeros.

What happens when we multiply 1 x 10 ? We get 10 correct? We said that 1 is the same as 1.0. We do not always show the decimal point because we do not always "need" to. Where is the decimal in 10 ?

That is correct, it is directly after the 0. So, 10 is 10.0. When we multiply by 10, and powers of 10, we are moving the decimal point 1 place to the right for each power of 10.

Solve this problem:

415.72 x 10 = ?

If we follow along with what we just discussed, we can just move the decimal point 1 place to the right for our answer.

The answer is 4157.2

You can write out the whole math problem and solve it, if you like.

Solve this problem:

5,006.2 x 10,000 = ?

Make sure you do not confuse the comma and the decimal point!

The answer is 50,062,000. It is a coincidence that the end of our sentence is also the location of where our decimal would go if we wanted it, or needed it. The answer is Fifty-two million sixty-two thousand.

Seem easy? It is. Try a few problems.

1. 10.56 x 10 =____ 2. 1225.01 x 1000 =____ 3. 126.89876 x 100 =____

See? Easy.

So now comes the trickiest part of decimals in division.

Let's call this Rule 2A. It is directly related to Rule 2, but only needs to happen when we have a decimal in our divisor.

Rule 2A

If we have to move the decimal out of our divisor, to make our divisor a whole number, we need to make the same change to our dividend. So, if we multiply our divisor by 100, we need to multiply our dividend by 100.

Or, we can say, if we need to move our decimal 2 places in the divisor, we need to move our decimal 2 places in our dividend.

Example.

$$1.3\overline{)22.1}$$

We have a decimal in our divisor. Just 1. If we move the decimal to the right 1 place, the number becomes 13. If we move the decimal in our dividend, 1 place, 22.1 becomes 221.

The new problem becomes:

$$\begin{array}{r} . \\ 13\overline{)221.} \end{array}$$

Because the decimal is now located behind the 1 inside of the dividend, we made sure to transfer it directly above, into the QUOTIENT, or where our answer will be once we do the math.

Here are some exercises for practice.

1.	7.2 ÷ 8 =_____	2.	1.2 ÷ 6 =_____	3.	1.2 ÷ 2.0 =_____
4.	3.5 ÷ 7 =_____	5.	1.0 ÷ 2 =_____	6.	1.5 ÷ .5 =_____
7.	2.8 ÷ 7 =_____	8.	1.6 ÷ .2 =_____	9.	4.9 ÷ 7 =_____
10.	737.1 ÷ 81 =_____	11.	93 ÷ 62 =_____	12.	112.2 ÷ 17 =_____
13.	362.6 ÷ 74 =_____	14.	86.4 ÷ 18 +_____	15.	161.7 ÷ 2.1 =_____
16.	381.3 ÷ 4.10 =_____	17.	157.5 ÷ 3.5 =_____	18.	160.2 ÷ 18 =_____
19.	817 ÷ 95 =_____	20.	112.1 ÷ 1.90 =_____	21.	199.8 ÷ 74 =_____
22.	73.71 ÷ 8.10 =_____	23.	9.30 ÷ 62 =_____	24.	11.22 ÷ 17 =_____
25.	36.26 ÷ 74 =_____	26.	56.36 ÷ 5.6 =_____	27.	15.75 ÷ .35 =_____
28.	7492.88 ÷ 81.8 =_____	29.	1181.4 ÷ 179 =_____	30.	891.21 ÷ 18.3 =_____
31.	7.371 ÷ 8.1 =_____	32.	.930 ÷ 62.000 =_____	33.	1.575 ÷ .035 =___

Percentages

Now is a perfect opportunity to introduce percentages.

In math, a percentage is a number ratio that can be expressed as a fraction of 100. So what we are determining is “a part per hundred.” The word percentage literally means “out of one hundred.”

Working out a percentage is easy, if there are 100 individual “things” that make up the whole.

$1 is a great example. There are 100 ‘cents’ in a dollar right? A quarter is 25% of a dollar. A dime is 10% of a dollar, a nickel is 5% of a dollar, and a penny, or ‘cent’, is 1% of a dollar.

But what if what we need to calculate is more or less than 100?

One method we have available, to determine percentage, is to first determine what 1% of the total would be.

Let’s say we are buying a laptop for $750. There is a sale for 20% off! What will the price be after discount? First, we can figure out what would 1% of 750 be? We will need to DIVIDE 750 into 100 equal parts. Go ahead and work the problem in the space provided.

$$100\overline{)750}$$

We get 7.50 as an answer, which means that 1% of $750 is $7.50. Great, but that is only 1%. We need to know 20%. Next we MULTIPLY, 20 x 7.50 = ? (If you are uncertain how the answer is 7.50, we explain decimals in division in the next example)

$$\begin{array}{r} 7.50 \\ \underline{\times\ 20} \\ 000 \\ \underline{15000} \\ 150.00 \end{array}$$

20% of 750 is 150. Now we would SUBTRACT the discount value from our original price.

$$\begin{array}{r} 750 \\ -\underline{150} \\ 600 \end{array}$$

The new price, after discount will be $600!

The next option we have for figuring percentages is based on truly understanding what is happening when we perform the math. Look at our previous example.

When we divided 750 by 100, what did our decimal point do?

Remember the number 750 is the same as 750.0. When we have a whole number, the decimal point is there, even if we are not actively using it.

When we determined what 1% of 750 was, it equaled 7.50. So, what happened to our decimal point?

Yes, we moved it over. How far did we move it over? We moved it over 2 spaces. How many zeros are there in the number one hundred? Two.

What direction did we move the decimal places? To the LEFT, because we DIVIDED the number. If we had MULTIPLIED, then we would have moved the decimal point to the RIGHT, like we previously discussed.

This "trick" works easily with values of 10, 100, 1,000, any power of 10. Just count the zeros, and move the decimal point the proper direction, matching the number of zeros.

10 x 10 = 100	10 x 100 = 1,000	100 x 100 = 10,000	1,000 ÷ 10 = 100
10,000 ÷ 10 = 1,000	850 ÷ 10 = 85	46.8 ÷ 100 = 0.468	98.6 x 100 = 9860

Can you see the pattern? This is always true, and you can use this truth to your benefit.

What else do we need to know about percentage?

If we want to know what the percentage of one number is, as it is related to another number is fairly simple to determine.

Let's say we are saving our money in order to buy that $750 laptop computer. We have saved for a few weeks, and now have $85. What percentage of $750 have we saved?

We will DIVIDE our $85 by the total amount of what we need, $750.

$$750 \overline{)\ 85}$$

Wait! Can we divide 750 into 85? 750 is a lot larger than 85! The answer is, Yes we can.

We can add zeros as we need to, in order to get a number "big enough" to divide into.

Remember that 85 is also 85.0. It is also 85.00 or 85.000 or 85.0000. Where we put the decimal in our answer is important. Let's look at the problem from the beginning, but we will use the decimal this time.

We know that 750 cannot be divided into 85.

But if we "add" a zero, our 85 becomes 85.0. We have not changed the value of our number. But we now have a number that 750 can begin to divide into. We must make sure to locate the decimal point on our answer line, directly above where it is in our math problem. We are just going to ignore the decimal, until we need our final answer.

```
          .
750 ) 85.0
```

Showing our work.

```
       00.11333
750 ) 85.00000
      750
       1000
        750
        2500
        2250
         2500
         2250
          2500
           2250
            250     (We can see a repeating pattern!)
```

So, we continued to work out our problem, trying to get a final solution where everything divided out evenly. But, a pattern developed, and we can determine that it will continue to repeat. We can stop at this point.

So we now have a decimal result. To find what percentage 85 is of 750, we have to multiply the decimal by 100.

0.11333 x 100 = ?

Remember, we do not have to work this out, we can just move the decimal point the same number of zeros.

11.333 %

In most instances, unless otherwise told to do so, we can "round" our number to the nearest 100th. Locate the number in the hundredth position, and then look at the next number. Round up or down as required.

We have saved 11.33% of our total so far. Only 88.67% to go!

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Let's do a few more percentages.

1.	50% of 60 =_____	2.	40% of 40 =_____	3.	10% of 50 =_____
4.	10% of 40 =_____	5.	60% of 50 =_____	6.	30% of 60 =_____
7.	10% of 30 =_____	8.	40% of 30 =_____	9.	20% of 10 =_____
10.	16% of 90 =_____	11.	13% of 60 =_____	12.	2% of 70 =_____
13.	16% of 70 =_____	14.	9% of 80 =_____	15.	15% of 20 =_____
16.	18% of 30 =_____	17.	80% of 27 =_____	18.	60% of 17 =_____
19.	10% of 22 =_____	20.	60% of 8 =_____	21.	70% of 24 =_____
22.	24% of 27 =_____	23.	3% of 18 =_____	24.	18% of 11 =_____
25.	19% of 8 =_____	26.	37% of 82 =_____	27.	17% of 7 =_____

LIFE HINT.

When you are out at a restaurant, and it is time for the tipping the wait staff because they have taken good care of you and your family.

Look at the TOTAL of your food bill.
It is up to you if you want to tip based on the before tax or after-tax amount.

Find the total, and simply move your decimal 1 place to the left. You now have 10% of the bill calculated. Want to leave 20% ? Multiply your original 10% by 2. Want to leave 15% ? Divide the 10 % by 2, and then add that to the original 10 %.

Fractions

Fractions are very interesting. Many people get really worried when they hear the word "fraction." It sometimes conjures up bad memories from our earlier school days. When we speak to many people about math, they are quick to say that they "do well with everything but fractions!"

Well, if you can do addition, subtraction, multiplication, and division, then you can do fractions!

A fraction is used to represent pieces of the whole. Remember in decimals, we told you that any of the numbers to the *right* of the decimal point represent a *portion* of a whole number? A fraction is the same thing, just written differently. Fractions are written as a ratio, or a relationship.

For example:

3/4 is a fraction. With words we would say this fraction as "three-fourths", or "three-quarters".

We have 3 out of 4 pieces, or we have 3 of 4 quarters.

For most references 4 quarters of something, will give us 1 whole something, just as in money, 4 quarters will equal a dollar.

4 x .25 = 1.00

1/4 + 1/4 + 1/4 + 1/4 = 1

Two halves will also give us 1 whole, just as two 50 cent pieces will give us a dollar.

1/2 + 1/2 = 1

2 x .50 = 1.00

From what we learned in our percentages section, we can now show that 3/4 is the same as 75%.
Did you realize that 3/4 is a division problem as well? It looks just like some of division problems from the previous section. So let's treat 3/4 as a division problem.

If we work out the math and 3 divide by 4, we would get .75.

We can then multiply that answer by 100, to get the percent.

Fractions are everywhere around us. We just need to get comfortable with them.

Fraction Terms

There are a few important terms to know and remember related to fractions.

We have the **Numerator**, **Denominator**, and **Vinculum**. The most important two are the numerator and denominator.

When we see a fraction written out, it is represented as one number over the top of another number.

$$\frac{3}{4} \quad \begin{array}{l}\text{(Numerator)}\\\text{(Denominator)}\end{array}$$

The number on the TOP is the **Numerator**. This represents how parts we actually have.

The number on the BOTTOM is the **Denominator**. This number represents how many parts are possible.

When written from side to side, the numerator occurs first, and the denominator second.

3/4

The funny word, that you may have never heard of before, **Vinculum**. That is the "bar" between the numerator and denominator. Remember this term, it may help you on trivia night!

A Few More Terms

Equation – a statement that the values of two mathematical expressions are equal, as shown by the = sign. E.g. $A^2 + B^2 = C^2$ and $1/2 = .50$.

Equivalent Fraction – fractions that represent the same value, ratio, relationship. When simplified the fractions will match.

Greatest Common Factor (GCF) – the largest factor that all denominators share.

Improper Fraction – a fraction in which the numerator is greater than the denominator.

Integer – a number that is not a fraction, a whole number.

Lowest (Least) **Common Denominator (LCD)** – the smallest number of all the common multiples of the denominators when two or more fractions are given. Used with addition and subtraction of fractions.

Mixed Number – a number consisting of an integer and a proper fraction.

Proper Fraction – a fraction that is less than one, the numerator is less than the denominator.

Simplest Terms – when we can not divide the numerator or denominator any further, in equal amounts, and still have whole numbers for the numerator and denominator.

Addition of Fractions

When we perform addition of fractions (or subtraction), we have one very simple rule. We must add apples to apples, and oranges to oranges. We must add LIKE TERMS. We do this by making sure that we have a common denominator. The denominator must be the same in order to add (or subtract) fractions.

On our first page of this section, we showed that four quarters equals one.

1/4 + 1/4 + 1/4 + 1/4 = 1

All of the denominators are the same, so we are permitted to add the fractions.

When adding fractions (once we have matching denominators), the denominator will stay the same in our final answer. We will only add the numerators (top number) together. In our example, if we do this we will get:

$$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{4}{4}$$

* Remember: The denominator stays the same in addition and subtraction of fractions.

At the beginning of this chapter, we mentioned that fractions are division problems.

What do we get when we divide 4 by 4?

That is correct, the answer is 1. We would write our 4/4 fraction as 1, because this is the SIMPLEST TERM for the problem. Hence 1/4 + 1/4 + 1/4 + 1/4 = 1.

What would the answer be for this problem?

1/5 + 1/5 = ?

Both fractions have the same denominator, so we can add the numerators, and we get:

1/5 + 1/5 = 2/5

And 2/5 is the simplest terms for the problem because we cannot divide anything into the numerator or denominator (evenly), they are as low as they can be.

Now, we will have fractions that do NOT have the same denominator. We will be *required* to make the denominators match before we can add (or subtract) them. Let's look at this example.

1/4 + 1/2 = ?

Mentally, we may see that we are adding one-quarter to one-half, and we would get three-quarters.

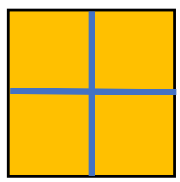
But we need to know how and why the answer is three-quarters.

$$1/4 + 1/2 = ?$$

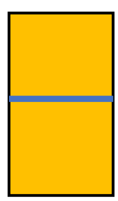
The denominators do not match. We need to find a common denominator. We need to find a number, or factor, that BOTH fractions have in common. One of our denominators is 2, and one of them is 4. We know that 2 x 2 = 4. We can use the 4, and we only need to adjust the second fraction to match the first fraction.

To do this, we must understand the idea of EQUIVALENT FRACTIONS. An equivalent fraction will have the same ratio, the same value as the original fraction.

Let's say there is a cake, and we cut it into four equal pieces.



We take two of the four pieces. Then we have 1/2 of the cake and leave 1/2 of the cake. We have two of the four pieces. There are two of the four pieces remaining.



We can see by the pictures that 2/4 pieces, is equal to 1/2 of the cake. 2/4 and 1/2 are EQUIVALENT FRACTIONS.

When we find a common denominator, we need to keep the ORIGINAL VALUE of the fraction. So, whatever we do to make the denominator equal to the number we have chosen, we need to do that same thing to the numerator. This will insure we have an equivalent fraction, or a fraction of the same value.

One method of figuring out common factors that the denominators share, is to count by increments of that number, and write each factor down. For example count by 5's. 5, 10, 15 , 20, 25, 30…. Or count by 3's: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30. Do this for each of the denominators until you find the SAME number in each factor list. Using the numbers 3 and 5, we see they share factors of 15, and 30. Since 15, is the smallest number they share in common, this would be considered the LOWEST (Least) COMMON DENOMINATOR. They also share 30 as a common factor.

Another method, is to simply multiply the two denominators together. 5 x 3 = 15. 15 is a common factor for 3 and for 5. In this example, it is a coincidence that when we multiply them together, we get the LOWEST COMMON DENOMINATOR. This is NOT always the case. See the next example.

5/12 + 7/15 = ?

List the multiples.

12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, 132, 144, 156, 168, 180

15: 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180

12 x 15 = 180

12 and 15 share 60, 120, and 180 as common factors. The LCD is 60. 120 and 180 are just common to both numbers.

Now that we have our common denominator (60), we can set up the problem.

Whatever we do to the bottom, we need to do to the top.

In order for 12 to become 60, we multiply it by 5. If we multiply the denominator (12) by 5, then we must multiply the numerator (5), by 5. We get 25/60.

In order for 15 to become 60, we multiply it 4. If we multiply the denominator (15) by 4, then we must multiply the numerator (7) by 4. We get 28/60.

Now our problem looks like this:

25/60 + 28/60 = ?

Add the numerators, and keep the denominator the same.

53/60

In this book, we are looking at simple math. We are only performing math problems with two fractions to add (or subtract). Sometimes there are math problems that have 3 or more fractions to be added together or subtracted from one another. You would be required to find the LCD for all the fractions. You would write a multiples list for all denominators to find the common factor(s).

Look at these examples and determine if they are equivalent fractions or not. Circle the problem if it is true.

1.	5/15 and 1/3	2.	4/6 and 8/10	3.	3/5 and 12/ 20
4.	2/18 and 1/9	5.	40/100 and 4/10	6.	27/45 and 9/15
7.	1 and 7/7	8.	13/26 and 1/3	9.	2 and 8/4

Did you like the those examples? Example 7 and example 9 were interesting! Let's discuss example 9 first.

In example 9, we see the fraction 8/4. This is called an **IMPROPER FRACTION.** An improper fraction has a numerator that is LARGER than the denominator. We may encounter improper fractions, and we may create a few when we are working to solve our math problems. The important thing is to know what they are, and that we can work with them. Unless told otherwise, we will always reduce our answers to simplest terms.

As mentioned before, fractions are division problems. In example 9, what do we get when we divide 8 by 4? That is correct. The answer is 2. Hopefully you said that 2 and 8/4 are equivalent fractions!

What is that you say? 2 is not a fraction? What if we wrote our 2 like this?

$$\frac{2}{1}$$

Is our 2 a fraction now? Yes, it is. It has the SAME VALUE as before, 2. Example question 7 also has a whole number, that can be converted to a fraction.

Whenever you have a number not written as a fraction, you can write it as one:

$$\frac{X}{1} \text{ (any number)}$$

ALL numbers can be fractions. Placing the number "over" 1, will show it as a fraction, that is of equal value to the original number.

When we have improper fractions, the numerator is LARGER than the denominator. In our example above, we also saw that fractions are division problems. When we encounter an improper fraction, we will need to SIMPLFY it, to make it a proper fraction. In our example, when we did the division, we saw that 8/4 = 2.

That was easy. It worked out exactly. But what if they do not work out exactly?

For example 7/6 = ?

It is still a division problem, and we can perform the math. And, since we are working with fractions, we do not need to work out all the decimal places!

$$
\begin{array}{rr@{\,}l}
 & & 1 \quad \text{(Whole Number)} \\
\text{(Divisor/ Dividend)} & 6 & \overline{)\,7} \\
 & & \underline{6} \\
 & & 1 \quad \text{(Remainder)}
\end{array}
$$

We see that 6 divides into 7, one time. If we worked the problem any further, we would begin to get decimals. 1 is our WHOLE NUMBER. We also have 1 “left over”, our REMAINDER. As long as our REMAINDER is LESS THAN our DIVISOR, we have performed the math correctly. Now, we write our whole number AND add our remainder over the denominator for our answer. We would write our answer out like this:

1 1/6.

We would say the answer as “one and one-sixth”.

Our answer is in simplest terms. When we have a whole number AND a fraction, we have what is called a **MIXED NUMBER.**

Let’s do another one.

Simplify. 13/3 =?

$$
\begin{array}{r@{\,}l}
 & \quad 4 \\
3 & \overline{)\,1\;3} \\
 & \underline{1\;2} \\
 & \quad 1
\end{array}
$$

Now we have our answer. 4 1/3 , or “four and one-third.”

Let’s do some problems. Simplify your answers.

1.	1/3 + 1/3 =_____	2.	2/8 + 2/8 = _____	3.	1/7 + 9/7 =_____
4.	8/5 + 9/5 =_____	5.	1/5 + 7/5 =_____	6.	4/3 + 1/3 =_____
7.	7/4 + 6/4 =_____	8.	6/2 + 8/2 =_____	9.	7/8 + 8/8 =_____
10.	1/10 + 1/7 =_____	11.	1/8 + 1/7 =_____	12.	1/9 + 1/9 =_____
13.	1/4 + 1/7 =_____	14.	1/6 + 1/6 =_____	15.	1/10 + 1/10 =_____
16.	1/9 + 1/5 =_____	17.	1/2 + 1/6 =_____	18.	1/9 + 1/10 =_____
19.	1/8 + 1/6 =_____	20.	2/7 + 5/8 =_____	21.	1/5 + 2/5 =_____
22.	2/6 + 2/6 =_____	23.	4/9 + 1/9 =_____	24.	1/7 + 2/9 =_____
25.	8/19 + 7/13 = _____	26.	1/16 + 1/14 =_____	27.	9/15 + 5/18 =_____
28.	1/10 + 7/9 =_____	29.	2/12 + 8/12 =_____	30.	6/13 + 1/9 =_____

By now, you should be feeling more comfortable with fractions. When we understand them, it is easier to work with them. A few of our examples included IMPROPER FRACTIONS, but as long as there is a common denominator, we can do the math, and then simplify our answer.

We recently discussed changing an IMPROPER FRACTION into a MIXED NUMBER. We discovered this is not difficult to do.

Sometimes, we need to add (or subtract) MIXED NUMBERS. For example:

1 1/2 + 1 1/2 = ?

This is a relatively easy problem, that many people can do “in their head.” Once again, we need to know how and why we get, the answer we get.

There are few different methods available to us.

The first method is to re-write the problem vertically, like we did with multiple digit math earlier in the book.

$$\begin{array}{r} 1\ \ 1/2 \\ \underline{+1\ \ 1/2} \\ ? \end{array}$$

We add the whole numbers first. Then add the fractions. Dealing with them separately.

$$\begin{array}{r} 1\ \ 1/2 \\ \underline{+1\ \ 1/2} \\ 2\ \ \ 2/2 \end{array}$$

We now have 2 for our whole numbers, and 2/2 for our fractions. If we simplify our fraction, what will we get ?

That is correct, we get 1. It is a whole number, and needs added to our whole numbers.

Our final answer is 3.

$$\begin{array}{c} 1\ \ 1/2 \\ \underline{+1\ \ 1/2} \\ 2 + 2/2 \\ 2 + 1 \\ 3 \end{array}$$

If the fraction portion of our numbers do not have matching denominators, we would need to find a common denominator for the fractions before we would be allowed to add them together. We do not have the adjust the whole numbers, only the fractions!

$$\begin{array}{rcr} 1\ \ 2/3 & = & 1\ \ 8/12 \\ \underline{+\ 2\ \ 1/4} & = & +\ \underline{2\ \ 3/12} \\ & & 3\ \ 11/12 \end{array}$$

Our answer is 3 11/12, Three and eleven-twelfths.

When we added the fractions together, their result was less than 1, because the numerator was smaller than the denominator (Proper Fraction). No adjustments are necessary. In our previous example, the fractions equaled 1 when they were added together. Remember to add any new whole numbers, to the whole numbers portion of the answer.

The second method we have to add (and subtract) MIXED NUMBERS, is to convert them into improper fractions, and then perform the math required to get our result.

We have already discussed how to convert improper fractions into mixed numbers. Now we have to go "the other direction."

Re-write the mixed number as an improper fraction.

1 6/13 = ?

To convert an improper fraction to a mixed number, we divided the denominator into the numerator first, and then wrote the whole number and any remainder over the denominator, as a fraction. Now we must first multiply the denominator by the whole number. Then add the numerator to that result, and write the result over the denominator.

1 6/13 = ?

13 x 1 = 13

13 + 6 = 19

19/13

Our improper fraction is nineteen-thirteenths.

Let's do a few more.

1.	1 5/11 =_____	2.	9 =_____	3.	5 1/3 =_____
4.	1 3/14 =_____	5.	2 3/8 =_____	6.	4 2/3 =_____
7.	1 3/11 =_____	8.	2 =_____	9.	1 2/11 =_____
10.	1 1/10 =_____	11.	1 7/11 =_____	12.	13/15 =_____

In the examples above, #2, #8 and #12 are NOT mixed numbers. #2 and #8 are whole numbers, and #12 is a proper fraction. They did not require any additional work.

Remember a mixed number is a whole number AND a proper fraction. A proper fraction is when the numerator is smaller than the denominator. An improper fraction has a numerator that is larger than the denominator. A whole number is the result of evenly dividing the numerator by the denominator, with no remainder.

Entrance Exam Tune Up

Now that we can work with mixed numbers and improper fractions, let's do some problems. Simplify your answers.

1.	1 4/6 + 2 1/4 =____	2.	2 1/5 + 1 =____	3.	2/4 + 2 2/3 =____
4.	2 + 2 2/3 =____	5.	1 4/5 + 1 3/6 =____	6.	3/4 + 2 2/4 =____
7.	5/6 + 2/6 =____	8.	1 2/6 + 1 2/3 =____	9.	5/6 + 1 2/6 =____
10.	2 3/4 + 3/4 =____	11.	2/5 + 4/6 =____	12.	1 3/4 + 1 =____
13.	1 6/12 + 1 8/9 =____	14.	10/11 + 1 1/9 =____	15.	1 9/10 + 1 3/11 =____
16.	1 2/8 + 2 =____	17.	2 2/5 + 2 2/7 =____	18.	3 1/5 + 3 =____
19.	1 7/10 + 1 6/11 =____	20.	2 6/7 + 2 2/9 =____	21.	1 2/9 + 2 =____
22.	1 1/12 + 10/12 =____	23.	1 5/11 + 1 5/8 =____	24.	2 + 2 1/9 =____
25.	1 2/11 + 1 4/12 =____	26.	2 1/9 + 1 2/9 =____	27.	1 6/9 + 1 4/7 =____
28.	10/6 + 9/4 =____	29.	11/5 + 6/6 =____	30.	2/4 + 8/3 =____
31.	4/2 + 8/3 =____	32.	8/2 + 10/2 =____	33.	9/5 + 9/6 =____
34.	12/3 + 12/4 =____	35.	3/4 + 10/4 =_____	36.	5/6 + 2/6 =____
37.	18/12 + 17/9 =____	38.	10/11 + 10/9 =____	39.	19/10 + 14/11 =____
40.	10/8 + 16/8 =____	41.	12/5 + 16/7 =____	42.	16/5 + 18/6 =____
43.	17/10 + 17/11 =____	44.	20/7 + 20/9 =____	45.	11/9 + 18/9 =____
46.	13/12 + 10/12 =____	47.	16/11 + 13/8 =____	48.	10/5 + 19/9 =____

Subtraction

Everything we explained for the addition of fractions also applies to the subtraction of fractions. Here are some exercises for practice. Simplify your answers.

1.	8/10 – 7/10 =____	2.	8/2 – 6/2 =____	3.	9/7 – 1/7 =____
4.	9/5 – 8/5 =____	5.	7/5 – 1/5 =____	6.	4/3 – 1/3 =____
7.	1/7 – 1/10 =____	8.	1/7 – 1/8 =____	9.	1/8 – 1/9 =____
10.	1/2 – 1/4 =____	11.	1/4 – 1/7 =____	12.	1/5 – 1/9 =____
13.	13/19 – 7/13 =____	14.	3/16 – 1/14 =____	15.	9/15 – 5/18 =____
16.	4/19 – 1/20 =____	17.	6/17 – 3/10 =____	18.	6/13 – 1/9 =____
19.	2 1/4 – 1 4/6 =____	20.	2 1/5 – 1 =____	21.	2 2/3 – 2/4 =____
22.	5 – 4 =____	23.	2 2/3 – 2 =____	24.	1 4/5 – 1 3/6 =____
25.	2 2/4 - 3/4 =____	26.	1 2/3 – 1 2/6 =____	27.	1 2/6 – 5/6 =____
28.	2 2/5 – 2 2/7 =____	29.	3 1/5 - 3 =____	30.	1 9/10 – 1 3/11 =____
31.	1 7/10 – 1 6/11 =____	32.	2 6/7 – 2 2/9 =____	33.	1 6/9 – 1 4/7 =____
34.	9/4 – 10/6 =____	35.	11/5 – 6/6 =____	36.	8/3 – 2/4 =____
37.	8/3 – 4/2 =____	38.	10/2 – 8/2 =____	39.	9/5 – 9/6 =____
40.	19/10 – 14/11 =____	41.	12/5 – 16/7 =____	42.	16/5 – 18/6 =____
43.	17/10 – 17/11 =____	44.	20/7 – 20/9 =____	45.	11/9 – 18/9 =____

Example 45 has a different answer than all our other examples. The answer is negative. We have focused on only positive answers up until this point. We just wanted you to see that you can do the math, even if it is a negative answer.

Entrance Exam Tune Up

Multiplication of Fractions

We just spent a large amount of time on addition and subtraction of fractions. We needed to make sure that when we finished, you would be more comfortable with fractions, that you would really understand them.

Multiplication of fractions is easy! Well "easier".

Remember when we told you that in order to add or subtract fractions, that they MUST have common denominators before we were allowed to do the math?

Multiplication of fractions does not have that rule. Multiplication of fractions has only 3 rules.

1. Multiply the numerators from each fraction.
2. Multiply the denominators from each fraction.
3. Simplify or reduce the answer.

Those are the three "Rules" for multiplying fractions.

We may be asked to multiply MIXED NUMBERS. We only have to convert the mixed numbers to fractions, and then follow our three rules.

Example.

	3/4 x 3/5 = ?
Multiply Numerators	3 x 3 = 9
Multiply Denominators	4 x 5 = 20
Answer	9/20
Simplify/ Reduce?	9/20 is the simplest form.

See how easy that was? Look at another.

	2 1/2 x 1 1/3 = ?
Change mixed number to fraction	5/2 x 4/3 = ?
Multiply numerators	5 x 4 = 20
Multiply Denominators	2 x 3 = 6
Answer	20/ 6
Simplify/ Reduce	3 2/6 = 3 1/3
Final Answer	3 1/3

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Multiplication Exercises

Simplify your answer.

1.	8/10 x 7/10 =____	2.	6/2 x 8/2 =____	3.	1/7 x 9/7 =____
4.	8/5 x 9/5 =____	5.	1/5 x 7/5 =____	6.	4/10 x 1/10 =____
7.	1/10 x 1/7 =____	8.	1/8 x 1/7 =____	9.	1/2 x 1/4 =____
10.	8/8 x 1/6 =____	11.	1/8 x 1/6 =____	12.	3/8 x 6/9 =____
13.	8/19 x 7/13 =____	14.	1/16 x 1/14 =____	15.	9/15 x 5/18 =____
16.	7/16 x 8/17 =____	17.	4/19 x 1/20 =____	18.	9/12 x 1/12 =____
19.	1 4/6 x 2 1/4 =____	20.	2/4 x 2 2/3 =____	21.	4 x 5 = ____
22.	1 4/5 x 1 3/6 =____	23.	1 2/6 x 1 2/3 =____	24.	2 3/4 x 3/4 =____
25.	2/5 x 4/6 =____	26.	2 x 2 2/4 =____	27.	1 4/5 x 4 =____
28.	1 6/12 x 1 8/9 =____	29.	10/11 x 1 1/9 =____	30.	2 2/5 x 2 2/7 =____
31.	10/6 x 9/4 =____	32.	12/3 x 12/4 =____	33.	11/4 x 4/4 =____
34.	18/12 x 17/9 =____	35.	20/7 x 20/9 =____	36.	13/12 x 10/12 =____

Division of Fractions

Multiplying fractions was a piece of cake. Once your problem is properly set up, there are only 3 simple rules to follow.

Division of Fractions has one extra "Rule". The rule requires us to do just 3 things. Ready for this?

Division Rule

Keep, Change, Flip.

That is the Division Rule. Sounds simple? It is.

But what does the division rule mean?

It means when solve our division problem, we are going to "Keep, Change, Flip."

Please follow along.

We have our division of fractions problem.

8/10 ÷ 7/10 = ?

Now we "Keep, Change, Flip" it.

Keep the first fraction. 8/10

We need to **Change** our sign (to multiplication). x

Then we **Flip** our second fraction. 10/7

Now we have a multiplication problem.

8/10 x 10/7 = ?

Multiply Numerators	8 x 10 = 80
Multiply Denominators	10 x 7 = 70
Answer	80/70
Simplify/ Reduce	1 10/70 = 1 1/7
Final Answer	1 1/7

Ohio Operating Engineers

Division Exercises

1.	6/2 ÷ 8/2 =_____	2.	8/5 ÷ 9/5 =_____	3.	1/7 ÷ 9/7 =_____
4.	2/ 8 ÷ 2/8 =_____	5.	9/10 ÷ 4/10 =_____	6.	1/9 ÷ 6/9 =_____
7.	1/10 ÷ 1/7 =_____	8.	1/8 ÷ 1/7 =_____	9.	1/8 ÷ 1/9 =_____
10.	1/2 ÷ 1/4 =_____	11.	1/9 ÷ 1/5 =_____	12.	1/9 ÷ 1/10 =_____
13.	8/9 ÷ 1/6 =_____	14.	1/8 ÷ 1/6 =_____	15.	2/7 ÷ 5/8 =_____
16.	1/7 ÷ 2/9 =_____	17.	8/19 ÷ 7/13 =_____	18.	1/16 ÷ 1/14 =_____
19.	9/15 ÷ 5/18 =_____	20.	2/8 ÷ 10/13 =_____	21.	2/12 ÷ 8/12 =_____
22.	4/19 ÷ 1/20 =_____	23.	6/17 ÷ 3/10 =_____	24.	3/17 ÷ 6/20 =_____
25.	1 4/6 ÷ 2 1/4 =_____	26.	2 ÷ 2 2/3 =_____	27.	1 4/5 ÷ 1 3/6 =_____
28.	1 2/6 ÷ 1 2/3 =_____	29.	1 ÷ 2 3/4 =_____	30.	2 3/4 ÷ 3/4 =_____
31.	1 9/10 ÷ 1 3/11 =_____	32.	2 2/5 ÷ 2 2/7 =_____	33.	2 ÷ 2 1/9 =_____
34.	10/6 ÷ 9/4 =_____	35.	8/2 ÷ 10/2 =_____	36.	9/5 ÷ 9/6 =_____

Mechanical Principles

Basically, mechanical ability means that you can understand mechanical principles, devices, tools, and the everyday physics that make them work. You also have the ability to reason and understand the direction of movement of gears in a system of gears. In addition, you can see the patterns of moving parts in engines and machines. Mechanical devices are an integral part of everyday life. When you imagine the numbers of cars on the highways, offices with machines and computers that make routine office duties easier, and the recreational vehicles used for vacations, you can quickly calculate that a person with mechanical abilities will have lots of work to do in a lifetime.

WHAT IS A MECHANICAL DEVICE?

A mechanical device is a tool invented to make a given task easier. For example, you could drive a nail into a piece of wood with a rock. However, a long time ago, a woodworker must have decided that there had to be a better way. A long slender handle with a hard piece of metal for striking the nail provided more accuracy and did not damage the wood as easily. Thus, the hammer was born.

Most mechanical devices were invented in the same manner—people looking for easier ways to perform their everyday jobs. Some mechanical devices— the lever, the wheel, and many hand tools—are thousands of years old. Other more complex devices, such as pumps and valves, were invented more recently. Many times, the idea of a new mechanical device exists but the technology to design it does not. For example, many years before the pump was invented, people probably discussed the need for an easier way to move water from the river to the town on the hill.

Mechanical devices cover a wide range and variety of tools. In general, mechanical devices are tools that relate to physical work and are governed by mechanical forces and movements. You can usually see what they do and how they work—as opposed to, say, a light switch or a battery, which are electrical devices. Some tools are used to directly accomplish a specific task, such as when you use a handsaw to cut a piece of wood. Others, such as pulleys and gears may be used indirectly to accomplish certain tasks that would be possible without the device but are easier with the device.

In your daily life, you see and use mechanical devices many times each day, so there’s no reason to be intimidated by a mechanical aptitude section on any exam.

The Mechanical Comprehension test administered as a portion of our entrance examination, is designed and intended to assess your logical reasoning, or thinking skills and performance. The exam has 36 questions of varying degrees and of various difficulty. It is a timed test, and you will have 20 minutes to answer all 36 questions.

If you go online and perform a search for Mechanical Comprehension and Mechanical Aptitude tests, there are literally thousands of examples available. Some are free, some are not. A very large portion will require your email and contact information, even though they are listed as “Free”.

This guide is a basic introduction to these types of questions, and an explanation of the principles related to the types of questions that may be asked.

Pulleys

A pulley consists of a wheel with a grooved rim in which a pulled rope or cable is run. Pulleys are commonly used with ropes or steel cables to change the direction of a pulling force. Pulleys are often used to lift things. For instance, a pulley could be attached to the ceiling of a room. A rope could be run from the floor, up through the pulley and back down to a box sitting on the floor. The pulley would allow you to pull down on the rope and cause the box to go up. That is, the pulley caused a change in direction of the pulling force. This is the principle behind the elevator.

Another common use for a pulley is to connect an electric motor to a mechanical device such as a pump. One pulley is placed on the shaft of the motor, and a second pulley is placed on the shaft of the pump. A belt is used to connect the two pulleys. When the motor is turned on, the first pulley rotates and causes the belt to rotate. That in turn causes the second pulley to rotate and turn the pump. This arrangement is very similar to a bicycle chain and gears. You may have seen pulleys used in a warehouse to lift heavy loads or on construction sites on cranes. The cable on a crane extends from the object being lifted up to the top of the crane boom, across a pulley and back down to the winch that is used to pull on the cable. In this situation the pulley again causes a change in direction of the pulling force from the downward force of the winch that pulls the cable to the upward movement of the object being lifted.

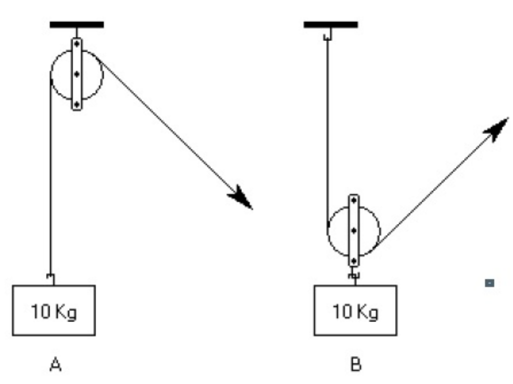
Single pulley questions are relatively straight forward. If the pulley is fixed, or unmoving, then the force required to lift the object, is equal to the weight of the object being lifted.

If the pulley moves with the object, then the force required will be equal to one-half of the weight.

The reason for this is, with a fixed pulley, there is only 1 line supporting the load.

With a movable pulley, there are 2 lines supporting the object. We divide the force required to lift the weight, by the number of lines that are supporting the object.

Which weight requires the least force to move?



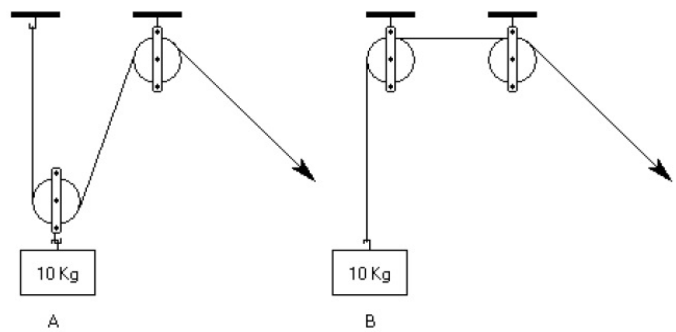
A) A B) B C) Both require the same force

In the example above, the left side is using a fixed pulley, which only has 1 section of rope supporting the weight. The right side is using a movable pulley, so it has 2 sections of rope supporting the weight.

The answer is B. We would need 5 Kg of force to move the 10 Kg weight.

There are two possible ways that two pulleys can be used. Either one pulley can be attached to the weight, or neither of them can be.

Which weight requires the least force to move?



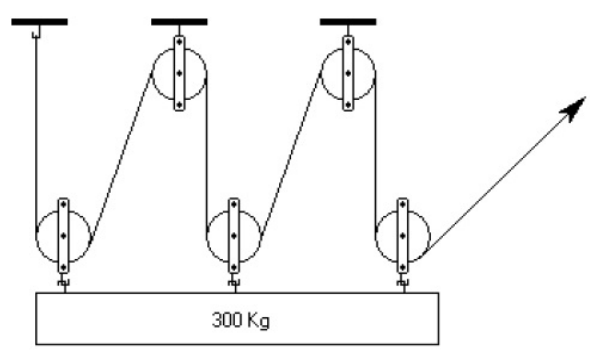
A) A B) B C) Both require the same force

In the above example, the image on the left has 1 fixed pulley, and 1 movable pulley. There are 2 sections of rope supporting the weight of the object. In the image on the right, both pulleys are fixed, so there is only 1 section of rope supporting the weight of the object.

The answer is A. The force required would be 5 Kg to move the 10 Kg weight.

It is possible to use more than two pulleys.

How much force is required to move the weight?



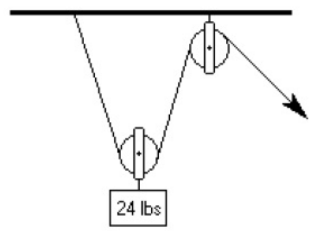
A) 100 Kg B) 150 Kg C) 50 Kg D) 60 Kg

In this example, we have 2 fixed pulleys, and 3 movable pulleys. If we count the sections of rope supporting the load, we would find that there are 6 sections. We need to divide the weight of the object by the number of sections of rope to determine how much force would be required to move the weight.

The answer is C. The force required would be 50 Kg.

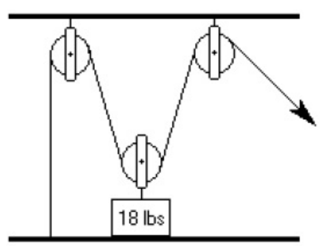
Answer the next several questions for pulleys.

Approximately how much force is needed to lift the weight?



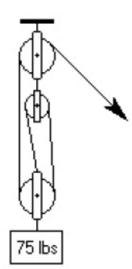
A	B	C	D	E
24 lbs	10 lbs	48 lbs	12 lbs	18 lbs

Approximately how much force is needed to lift the weight?



A	B	C	D	E
36 lbs	10 lbs	18 lbs	9 lbs	14 lbs

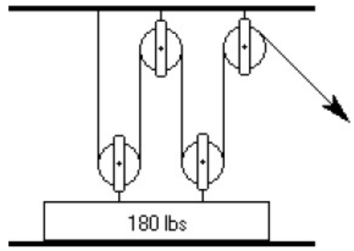
Approximately how much force is needed to lift the weight?



A	B	C	D	E
75 lbs	35.5 lbs	25 lbs	50 lbs	15 lbs

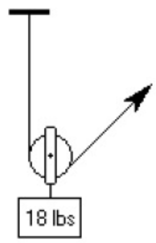
Entrance Exam Tune Up

Approximately how much force is needed to lift the weight?



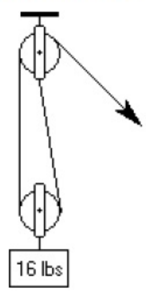
A	B	C	D	E
30 lbs	45 lbs	60 lbs	90 lbs	120 lbs

Approximately how much force is needed to lift the weight?



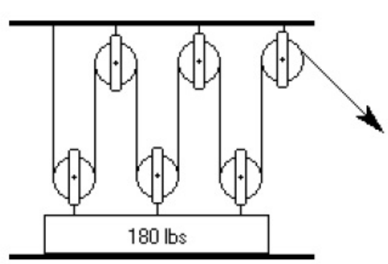
A	B	C	D	E
9 lbs	18 lbs	6 lbs	24 lbs	10 lbs

Approximately how much force is needed to lift the weight?



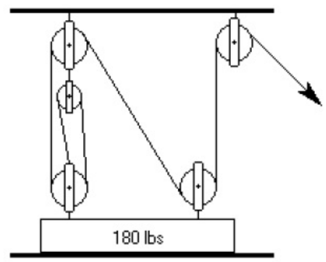
A	B	C	D	E
9 lbs	8 lbs	6 lbs	4 lbs	16 lbs

Approximately how much force is needed to lift the weight?



A	B	C	D	E
15 lbs	30 lbs	45 lbs	60 lbs	90 lbs

Approximately how much force is needed to lift the weight?



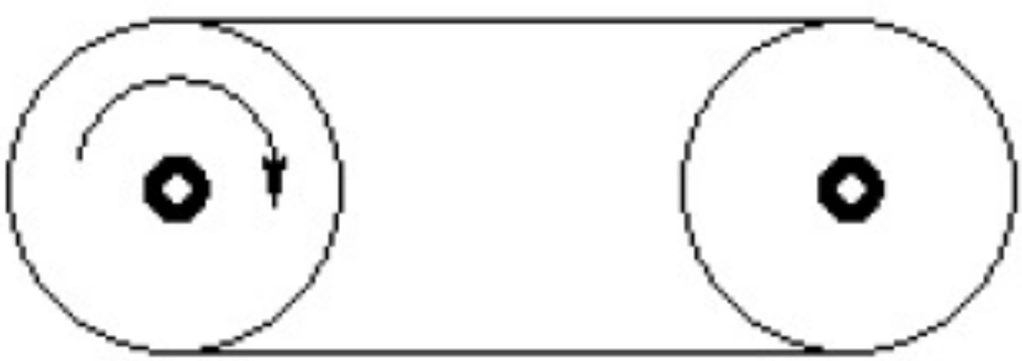
A	B	C	D	E
30 lbs	36 lbs	45 lbs	60 lbs	90 lbs

Gears

A gear is a wheel or cylinder with "teeth" that meshes with another toothed component to transmit motion, or to change speed or direction. Gears are attached to a rotating shaft turned by an external force like an electric motor or an internal combustion engine. Gear are used in many mechanical devices including automotive transmissions, bicycles, and carnival rides such as Ferris wheels and merry-go-rounds.

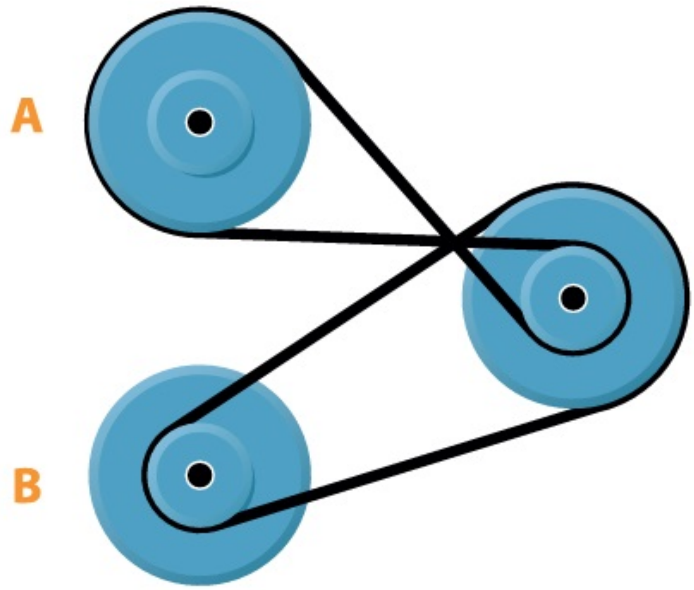
Gears can be used in several different configurations. Two gears may be connected by directly touching each other as in an automotive transmission. In this arrangement, one gear spins clockwise and the other rotates counter-clockwise. Another possible configuration is to have two gears connected by a loop of chain as on a bicycle. In this arrangement, the first gear rotates in one direction causing the chain to move. Since the chain is directly connected to the second gear, the second gear will immediately begin to rotate in the same direction as the first gear.

Many times, a system will use two gears of different sizes as on a ten-speed bicycle. This will allow changes in speed of the bicycle or machine. Problems about gears will always involve rotation or spinning. The easiest way to approach test questions that involve gears is to draw a diagram of what the question is describing. Use arrows next to each gear to indicate which direction (clockwise or counter-clockwise) it is rotating.

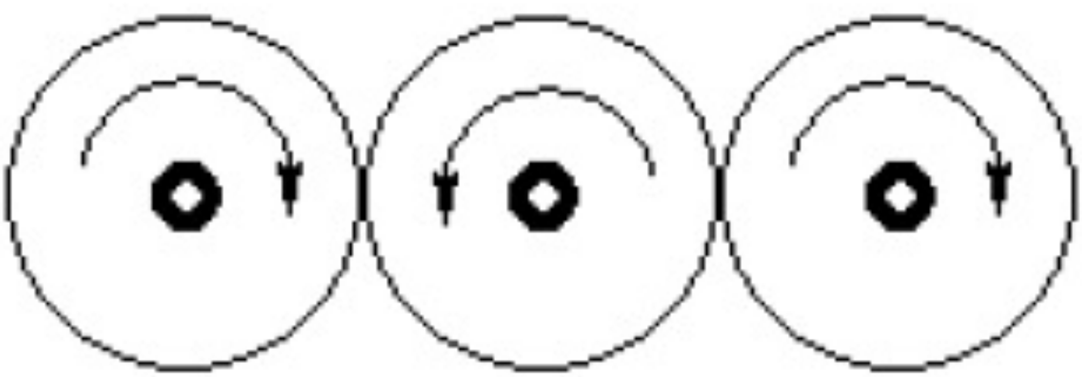


Gears Connected by Belt

If the belts are shown twisted, this indicates a direction change. If the pulley on the right turns, which of the wheels will turn in the same direction?



If the gears are touching (meshed), then adjacent gears move in opposite directions. In this image, the first and third gear will turn in the same direction. When there are an odd number of meshed gears, the last gear will always turn in the same direction as the first one. An even number of gears will result in turning opposite directions.

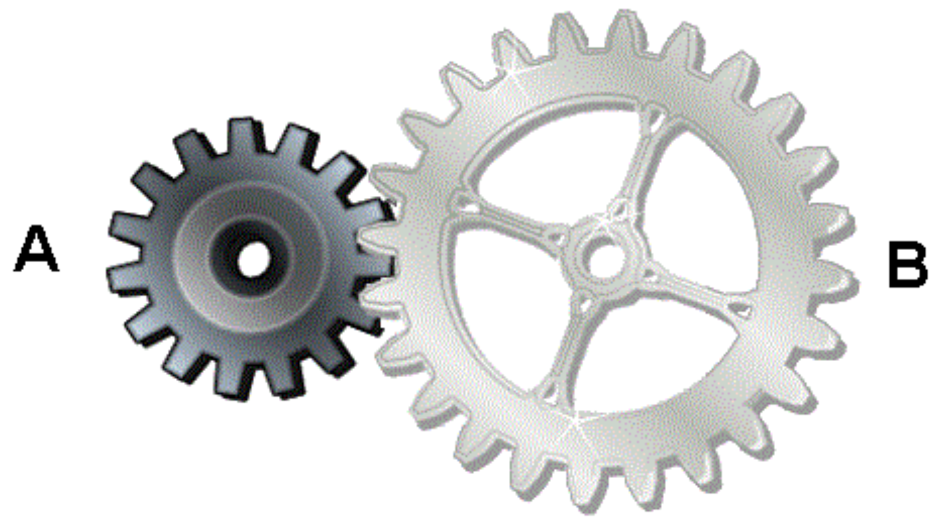


Gears Meshed

As mentioned, there are many gear questions that will ask about the speed of the gears. In general, a large gear will turn the slowest and the small gear turns fastest.

When you are presented with gears, the size order will be small to large (as seen below) or large to small. This may be repeated or alternated in sequence.

When the small gear is first, and the large gear is second, a speed reduction is indicated. And it is the opposite when the large gear is shown first, and the small gear is shown second.



Answer the following questions for gears and pulleys.

If gear X turns clockwise at a constant speed of 10 rpm. How does gear Y turn?

A	B	C	D	E
anti c/w 10 rpm	c/w 10 rpm	c/w 5 rpm	anti c/w 5 rpm	c/w 20 rpm

Entrance Exam Tune Up

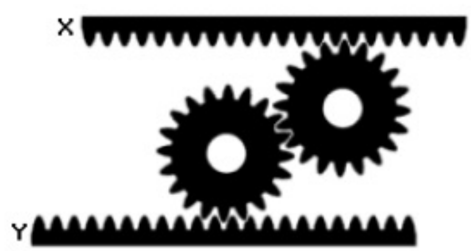
If gear X turns clockwise at a constant speed of 10 rpm. How does gear Y turn?

A	B	C	D	E
anti c/w 10 rpm	c/w 10 rpm	c/w 5 rpm	anti c/w 5 rpm	c/w 20 rpm

If gear X turns clockwise at a constant speed of 10 rpm. How does gear Y turn?

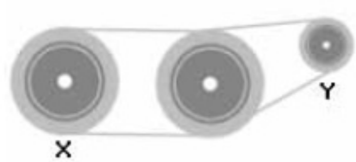
A	B	C	D	E
anti c/w 10 rpm	c/w 10 rpm	c/w 5 rpm	anti c/w 5 rpm	c/w 20 rpm

If bar Y moves left a constant speed. How does bar X move?



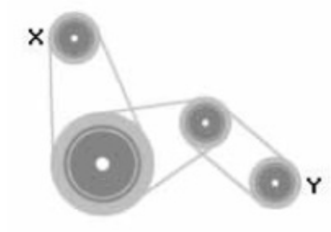
A	B	C	D	E
Left, Faster	Left, Same	Left, Slower	Right, Same	Right, Slower

If drive wheel X rotates clockwise at a speed of 10 rpm. How does wheel Y turn?



A	B	C	D	E
anti c/w faster	c/w slower	c/w faster	anti c/w slower	anti c/w same

If drive wheel X rotates clockwise at a speed of 10 rpm. How does wheel Y turn?



A	B	C	D	E
anti c/w faster	c/w slower	c/w faster	anti c/w slower	c/w same

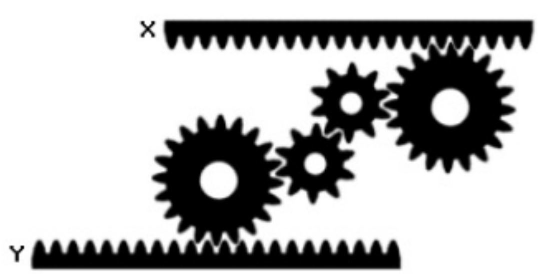
If gear X turns clockwise at a constant speed of 10 rpm. How does gear Y turn?

A	B	C	D	E
anti c/w 10 rpm	c/w 10 rpm	c/w 20 rpm	anti c/w 5 rpm	anti c/w 20 rpm

If gear X turns clockwise at a constant speed of 10 rpm. How does gear Y turn?

A	B	C	D	E
anti c/w 10 rpm	c/w 10 rpm	c/w 5 rpm	anti c/w 5 rpm	c/w 20 rpm

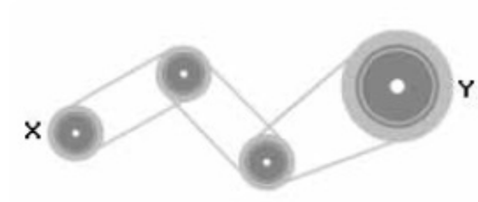
If bar Y moves left a constant speed. How does bar X move?



A	B	C	D	E
Left, Faster	Right, Same	Left, Slower	Left, Same	Right, Slower

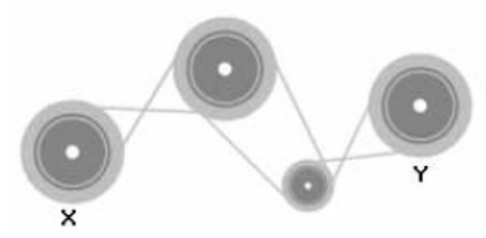
Entrance Exam Tune Up

If drive wheel X rotates clockwise at a speed of 10 rpm. How does wheel Y turn?



A	B	C	D	E
anti c/w faster	c/w slower	c/w faster	anti c/w slower	anti c/w same

If drive wheel X rotates clockwise at a speed of 10 rpm. How does wheel Y turn?

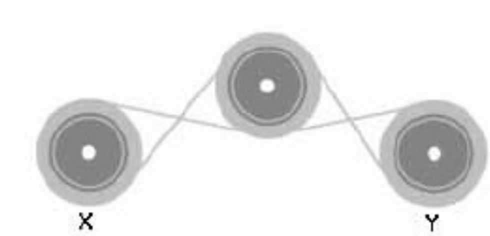


A	B	C	D	E
anti c/w faster	c/w slower	c/w faster	anti c/w slower	c/w same

If gear X turns clockwise at a constant speed of 10 rpm. How does gear Y turn?

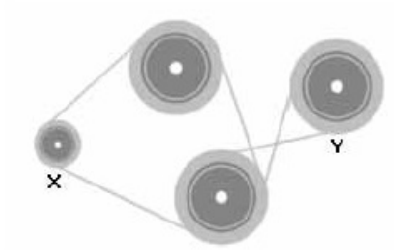
A	B	C	D	E
anti c/w 10 rpm	c/w 10 rpm	c/w 5 rpm	anti c/w 5 rpm	c/w 20 rpm

If drive wheel X rotates clockwise at a speed of 10 rpm. How does wheel Y turn?



A	B	C	D	E
anti c/w faster	c/w slower	c/w faster	anti c/w slower	c/w same

If drive wheel X rotates clockwise at a speed of 10 rpm. How does wheel Y turn?



A	B	C	D	E
anti c/w faster	c/w slower	c/w faster	anti c/w slower	c/w same

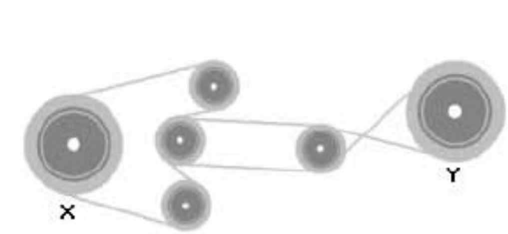
If gear X turns clockwise at a constant speed of 10 rpm. How does gear Y turn?

A	B	C	D	E
anti c/w 10 rpm	c/w 10 rpm	c/w 5 rpm	anti c/w 5 rpm	c/w 20 rpm

If gear X turns clockwise at a constant speed of 10 rpm. How does gear Y turn?

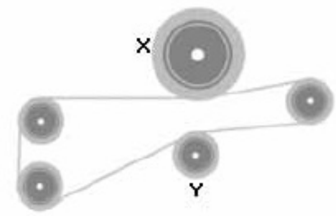
A	B	C	D	E
anti c/w 10 rpm	c/w 10 rpm	c/w 5 rpm	anti c/w 5 rpm	anti c/w 20 rpm

If drive wheel X rotates clockwise at a speed of 10 rpm. How does wheel Y turn?



A	B	C	D	E
anti c/w faster	c/w slower	c/w faster	anti c/w slower	c/w same

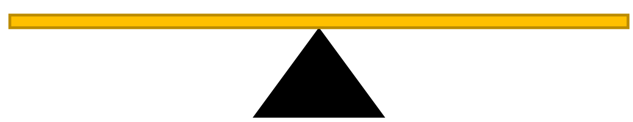
If drive wheel X rotates clockwise at a speed of 10 rpm. How does wheel Y turn?



A	B	C	D	E
anti c/w faster	c/w slower	c/w faster	anti c/w slower	c/w same

<u>LEVERS</u>

A lever is a very old mechanical device. A lever typically consists of a metal or wooden bar that pivots on a fixed point. The object of using a lever is to gain a mechanical advantage. Mechanical advantage results when you use a mechanical device to make a task easier; that is, you gain an advantage by using a mechanical device. A lever allows you to complete a task, typically lifting, which would be more difficult or impossible without the lever.



The most common example of a lever, is a playground see-saw. A force (a person's weight) is applied to one side of the lever, which causes the weight on the other side (the other person) to be lifted. However, since the pivot point on a see-saw is in the center, each person must weigh the same or things do not work well. You see, a see-saw is a lever with no mechanical advantage. If you push down on one side with a weight of ten pounds you can only lift a maximum of 10 pounds on the other side. This is no great advantage.

A lever amplifies input force to provide a greater output force, which is said to "provide leverage."

Another example of a simple machine, that happens to also be a lever, is the catapult. A basic catapult is a beam (lever), propped by a fulcrum (pivot point). The catapult magnifies force in order to throw an object! The longer the lever is, more potential for distance in throwing the object.

Some mechanical principle exams ask about catapults, because they are simple machines, and they utilize the leverage principle. Many would believe that questions like these should not be asked, because we do not use catapults anymore. Have you ever watched the summer Olympics? Or been to the swimming pool? Think about it! Catapults still exist, and we still use them, but for different purposes.

Center of Gravity

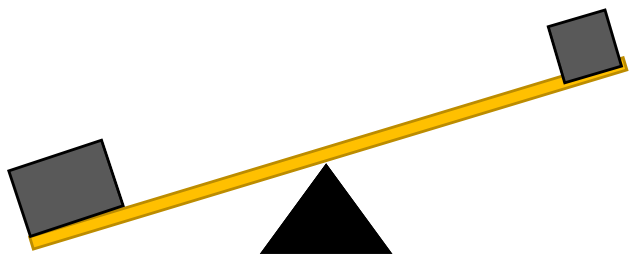
The center of gravity is a geometric property of any object. The center of gravity is the average location of the weight of an object. The weight of the object is evenly distributed around the center of gravity.

Think about our see-saw.

The point located in the center, where the board is balanced, is called the fulcrum. When the load is evenly distributed on both sides of the fulcrum, the load is balanced.

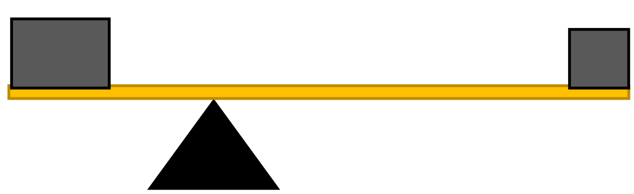
When the load is evenly balanced, the fulcrum point is exactly at the center of gravity.

What happens when we put two unevenly weighted items on each side of the fulcrum point?



As we can see in the illustration, the side that weighs more, is now on the ground, and the lighter side is now held in the air. The larger box is exerting more force on the board that was previously balanced on the fulcrum point. The weight is no longer evenly distributed on either side of our fulcrum point. There is more weight (force) on the left side.

We have two options available to balance our load. We can add or remove weight, until both sides are equal again, and then regain our balance. Or we can move our fulcrum point to where the weight is evenly distributed again, the center of gravity.

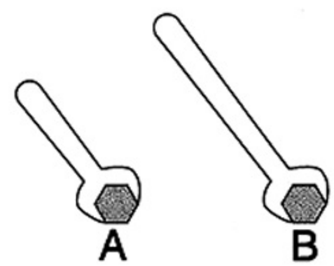


This is how we can play on the see-saw with different sized people. We move the fulcrum point closer to the larger person. The overall size difference between the people, will determine how far we need to move the fulcrum point.

When we move the fulcrum point closer to the larger weight, the amount of effort (force) required to raise the weight was reduced. We have increased the length of our lever, which amplifies our input (force). Now we have mechanical advantage!

If we had moved the fulcrum farther away from the load, and closer to the "effort" side, the amount of effort required to raise the load would have increased. Definitely not an advantage!

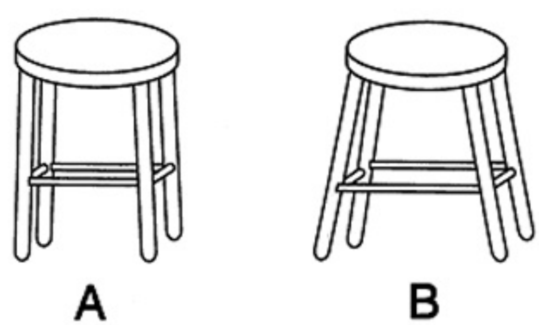
Using the idea of leverage, which bolt is easier to remove?

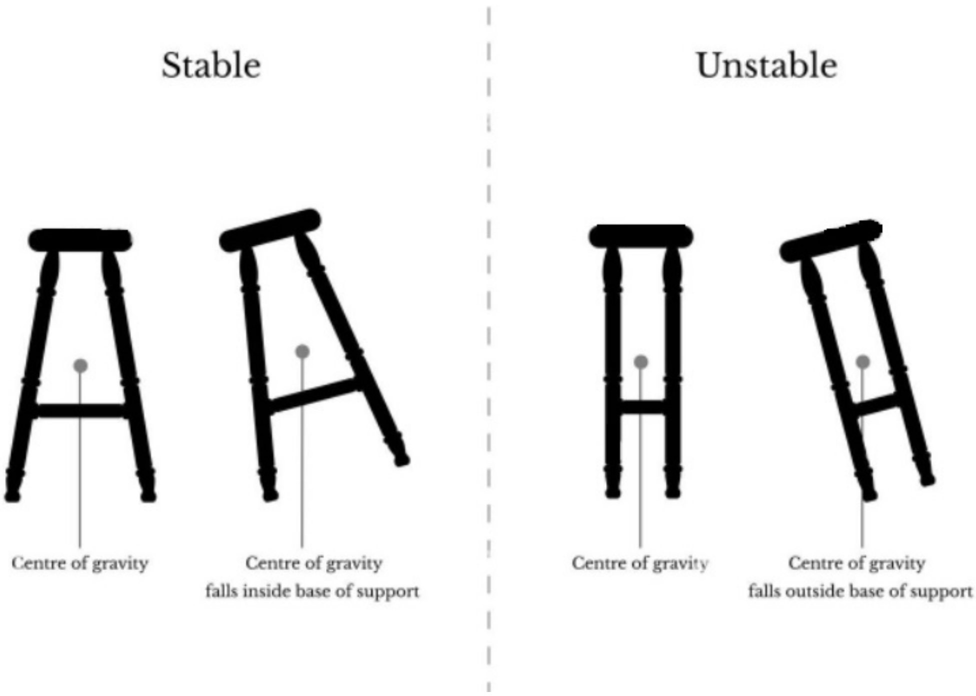


B is correct. The wrench moving bolt B is longer, and provides more leverage, thereby requiring less effort.

Stability

Which of these stools is more likely to tip over?

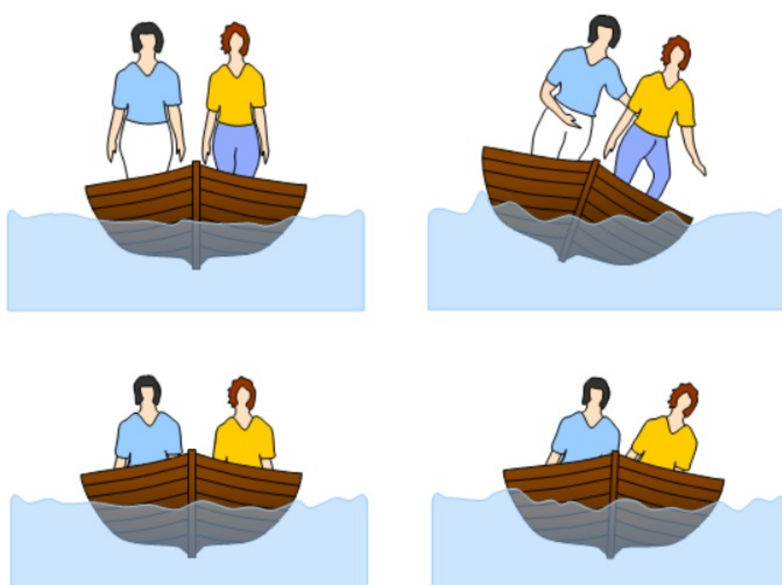




If you answered A, then you are correct. Stool A will tip over easier, because it has a narrower base. A narrow base will allow the center of gravity to travel outside the stability limits faster. Once the center of gravity moves outside of the area of stability, the object will tip over.

This is true for buildings, automobiles, and heavy equipment. The Center of Gravity is VERY important!

It also helps to know that a high center of gravity, as compared to a low center of gravity, will also allow objects to tip over easier.



This is why we have always been told that it is not a good idea to stand up in the boat, that we should sit down. The higher center of gravity will allow the center of gravity to travel outside of the area of stability easier.

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Strongest Shapes

If you take a moment and think about the many different shapes that are used in construction, you will begin to notice a pattern. There are certain shapes that are used repeatedly.

Arches

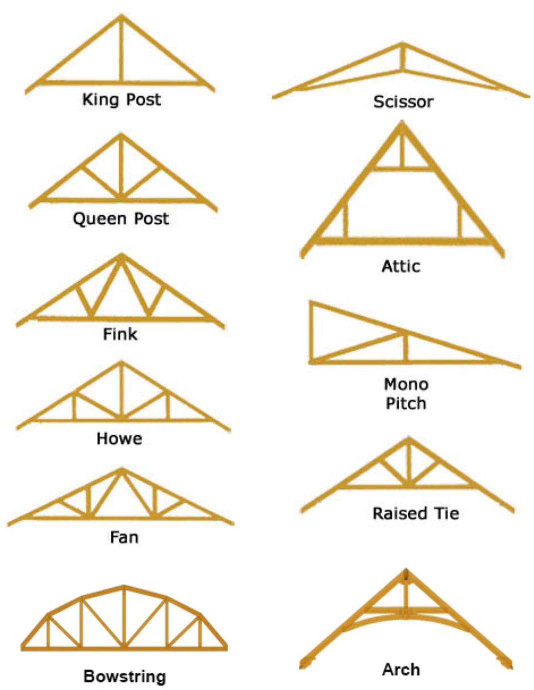
Do you remember where you have seen these shapes?

These are the Roman Aqueducts. These have been standing for centuries due to their strength. We still use these shapes in our building designs today.

Yes, even the hard hats we wear on the jobsite have an arch in their design, because of its strength.

Triangle

The shape of the triangle is very strong, as it holds its shape. The triangle is common for building supports and trusses.



Where else have you seen triangles in use within construction and design? In nature?

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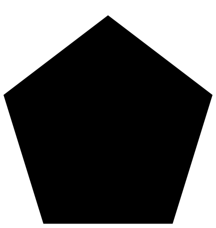
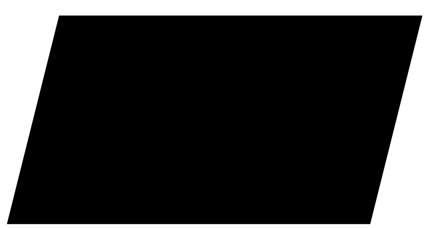
The Arch and the Triangle are two of the strongest shapes known to mankind. This is why they are chosen to be included in our structural design for strength and support.

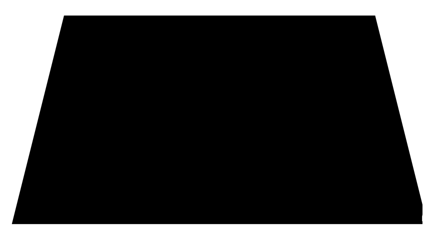
We see many other shapes in design, that are used more for decorative purposes, because they are not as strong. If the other shapes are used with the intent to support a load, the load is not intended to be very heavy, or critical.

The other very strong shape we see is a Hexagon. This shape is extremely strong. Can you think of where a hexagon is a natural shape? How is it built?

Other shapes that are sometimes used.







Pentagon

Parallelogram

Trapezoid

Water Towers

The primary function of water towers is to pressurize water for distribution. Elevating the water high above the pipes that distribute it throughout the surrounding building or community ensures that hydrostatic pressure, driven by gravity, forces the water down and through the system.

Only vertical height affects pressure! The size of the tank does not affect the pressure. The water demand influences the size of the water tank; a large tank will empty slower for a large flow demand.

There are many questions that can be asked related to the amount of pressure provided by water towers. In most of these problems, they will show you various sizes of water towers, with different capacities that they can hold. As stated above, the volume of water has nothing to do with the water pressure provided. The height of the water tower is the controlling factor for the amount of pressure provided.

Gas Pressure

Which one weighs more, a pound of feathers or a pound of bricks?

The answer is: They are equal! Although one brick will weigh more than one feather, the question is about the weight the two items. A pound is a pound, is a pound, is a pound!

What do bricks and feathers have to do with Air Pressure?

Unless you are going to be working with Boyle's Law, Charles's Law, The Ideal Gas Law, or some other calculations based upon atmospheric conditions, you need to understand the actual question that is asked.

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When dealing with gas pressure problems, you have to be given certain pieces of information.

The temperature of the gas, the volume of a gas, the pressure of a gas, or the quantity of the gas.

Depending on the information they give you, will determine what the testers are asking for.

Normally they will inform you if any condition is the same, or if any condition is different, or if any condition is changing.

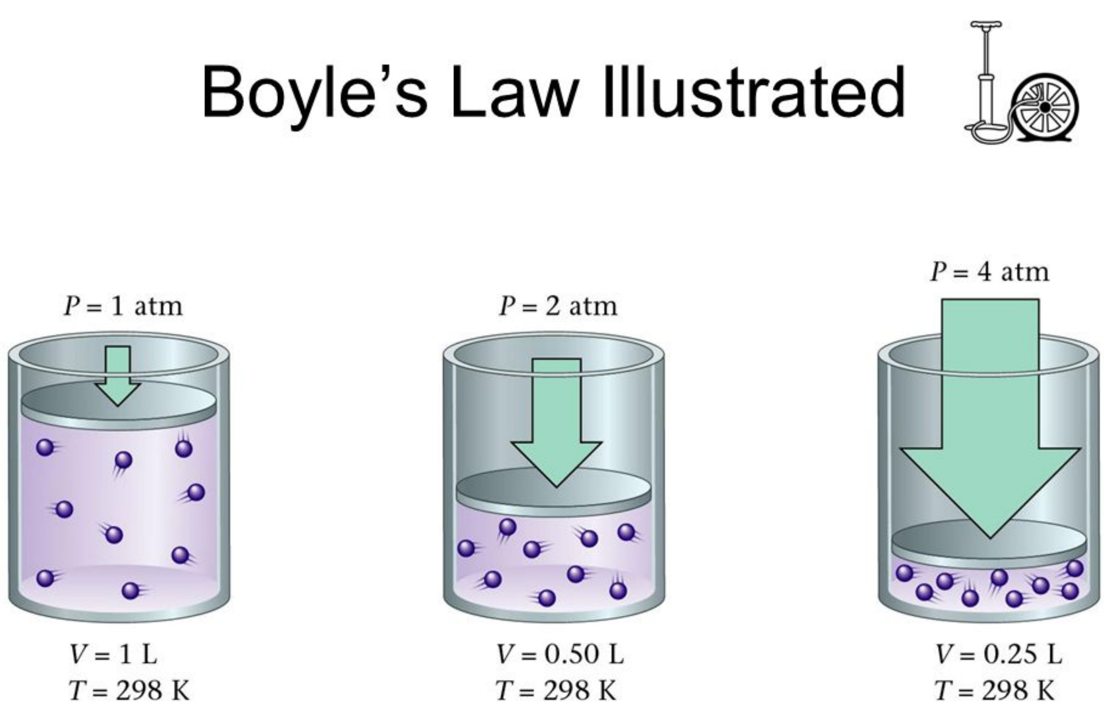
If you are interested in obtaining your Part 107 Remote Pilot's License to fly drones professionally, you will certainly learn about atmospheric air pressure, and pressure altitudes and changing air masses!

Here are a few gas laws to wrap your head around.

Boyle's Law

-gives the relationship between pressure of a gas and the volume of a gas at a constant temperature. The volume of gas is inversely proportional to the pressure of gas at a constant temperature.

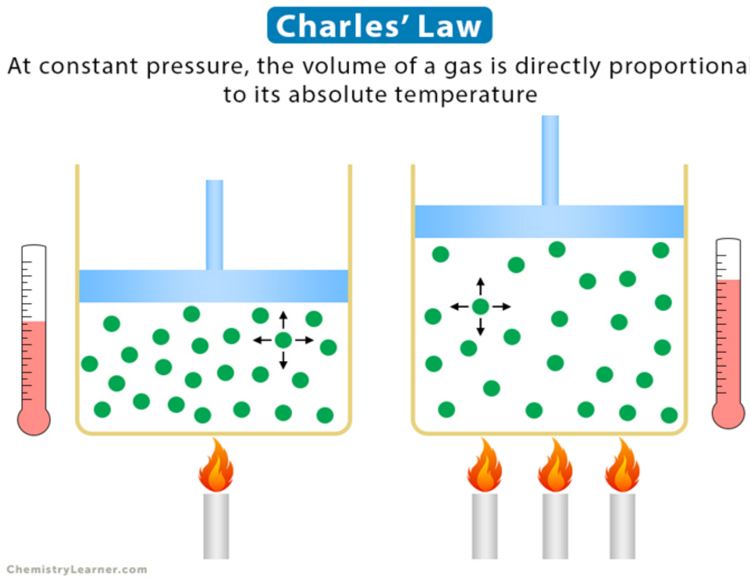
Basically, increase the volume (the space it occupies, not the amount), decrease the pressure. Decrease the volume (reduce the space it occupies, not the amount), increase the pressure.



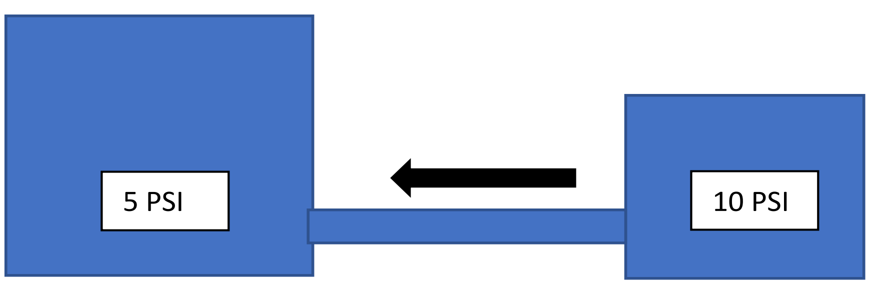
Charles's Law

-The volume of a gas with a fixed mass is linearly proportional to the temperature.

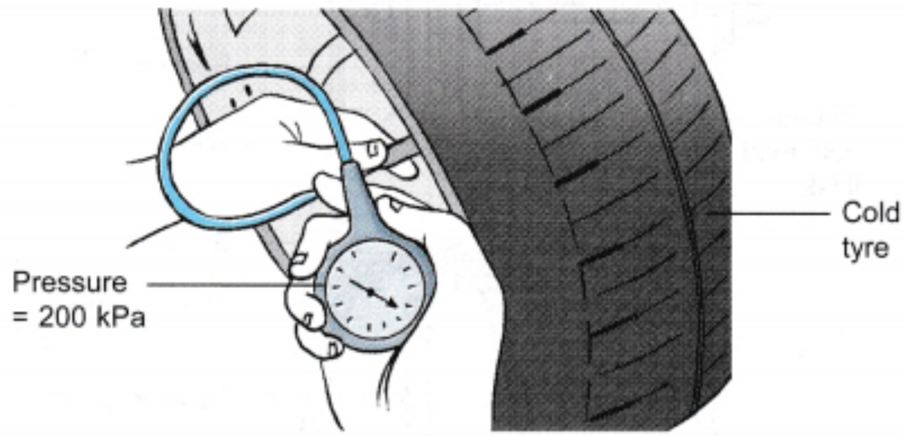
Basically, if the amount of the gas remains the same (quantity/ mass), increase the temperature and you will increase the volume (space it occupies). Reduce the temperature and reduce the volume (space it occupies). The pressure will remain the same.



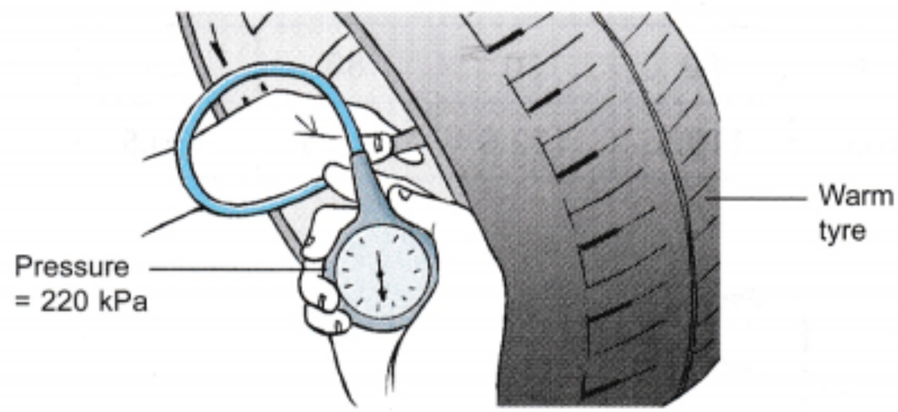
Gases and liquids both behave in a very similar manner. They are always trying to get to a state of equilibrium. Where everything is balanced. If you have higher pressure in one area, and lower pressure in another, the gas or liquid will flow to the lower pressure side until they are equal.



If you heat a gas or a liquid, you will cause an increase in temperature that will cause pressure and volume to increase. If you cool a gas or liquid, it will cause the pressure or volume to decrease.



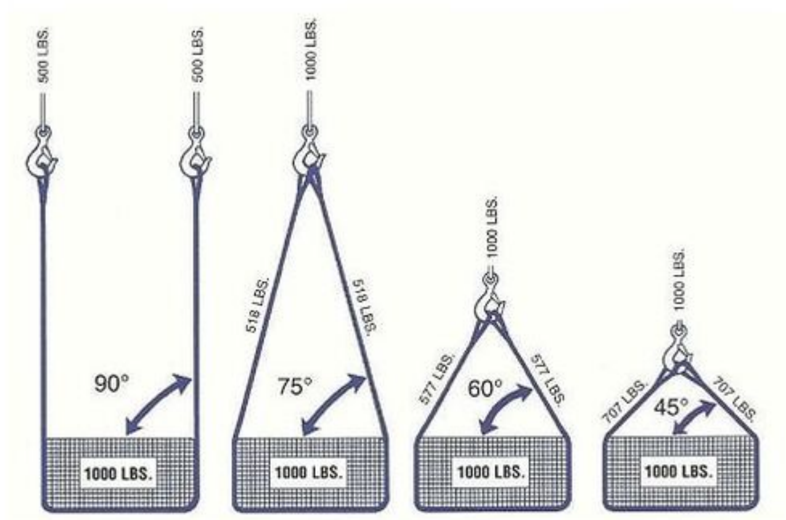
(a) *Before starting the journey*



(b) *After reaching the destination*

These are general observations. If you have studied or are working toward a deeper and greater understanding of chemistry and physics, these examples and explanations would seem to be oversimplified.

Lifting Loads with Rigging



Have you ever had to lift something into the air in order to move it? Normally we just wrap a strap, a rope, or a chain around it and lift it up. But, not understanding limitations to our lifting gear, can have serious consequences.

In the previous graphic, we can see that when using our "rigging", that the proper positioning and arrangement of that "rigging" is important. The smaller the angle between what we are lifting up and the point at which we are lifting it up, the greater the amount of stress we are placing on the "rigging" itself.

If we have an angle that is too small, we can cause a failure of the system. If this happens, you, someone you work with, or someone you care for, could be injured, or worse, killed.

We must always know the weight of the object we are lifting, and the capacity that our "rigging" can safely lift. If our "rigging" is not properly rated, it will fail. If we use proper rigging improperly, we can cause it to fail.

Tension

Tension is the act of straining or stretching. Applying a force to something, which tends to stretch it.

All physical objects that are in contact can exert forces on each other. We give these contact forces names based on the type of objects in contact. If one of the objects exerting the force happens to be a rope, string, chain, or cable, we call the force tension. We can even have tension created by pressure being exerted against a connection point in an architectural element. Think of a bolt connecting a steel beam to a column. Tension is created when there is a pulling or pushing action on that connection.

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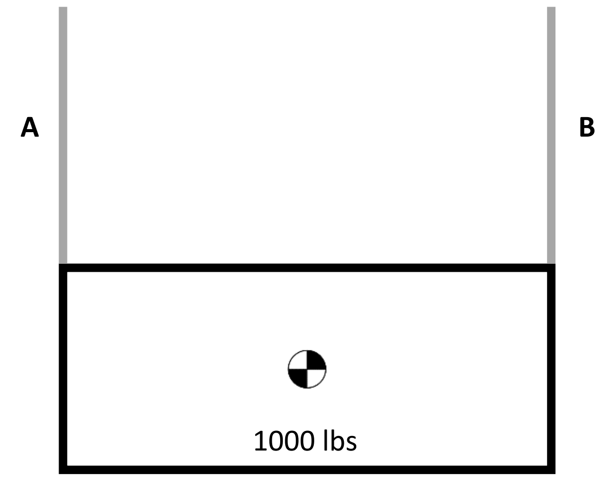
Look at this example:

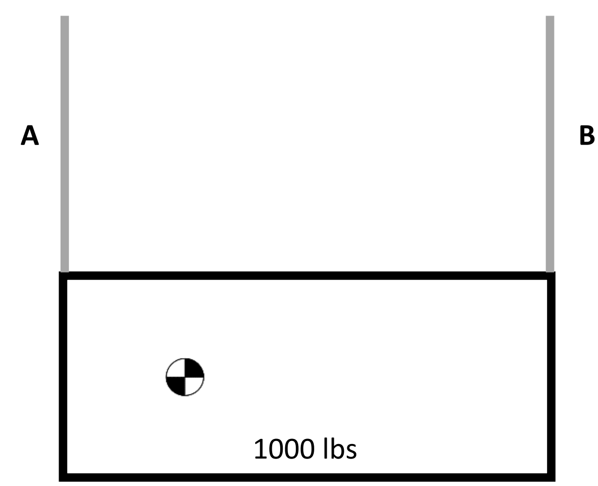


We are lifting a load of 1,000 lbs. We have two “ropes”, one holding each side. The ropes each have a capacity of 2,000 lbs. In this example, which rope is under more tension? If they are the same, answer C.

If answered C, and said that they are under an equal amount of tension, you are correct.

What if we made an adjustment? Which “rope” is under more tension?

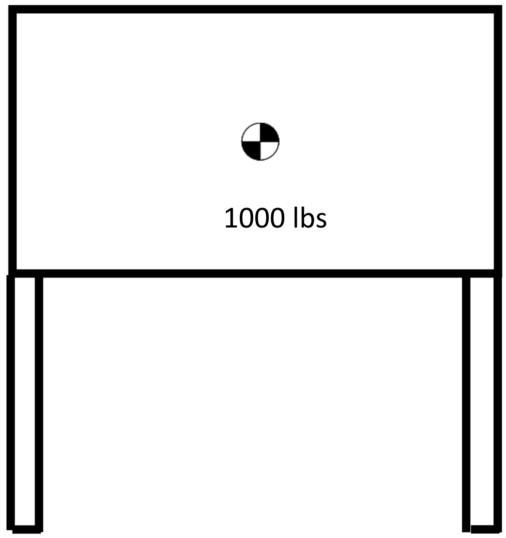
If they are the same, answer C.



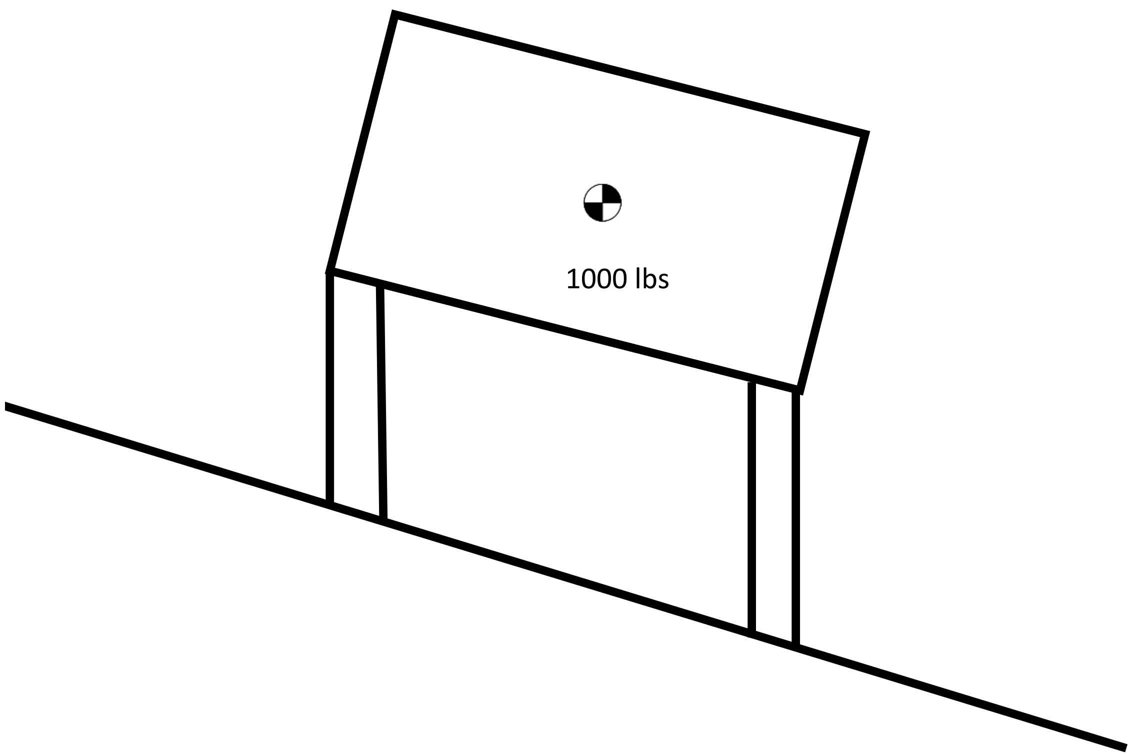
You are correct if you answered A.

What made the difference? That is correct, there is more weight on the A side, than there is on the B side.

If we were to reverse the image, and make the ropes that are suspending our load, into supporting columns or beams, the same principles would apply.



With the center of gravity of the load or structure exactly in the center of the load, all weight supported by the columns will be equal. If the center of gravity is located closer to one side or the other, then the side that is closest to the center of gravity would be supporting more of the load.



If we were to change the location of the supported structure to a surface that was not on a level plane, but sloped, the downhill, or lower column would support more of the load than the uphill column.

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Forces Applied to Structures

Structures are all around us. There are many considerations that go into the design of those structures. One of those considerations is understanding the forces that will act on the structures that we build. Prior to this section, we introduced the idea of strong shapes that are used in the design of our structures. We need our structures to be strong, because of the forces acting upon them. People, cars and trucks, windstorms, rainstorms, and flooding. All these examples, and more, will exert forces on the structures we build.

Compression: A pushing force that squeezes or compacts a material together.

Tension: A pulling force that stretches or pulls material apart.

Sheer: A force that tears or bends a material by pushing in parts of the material in opposite directions.

Torsion: A force that acts on material by twisting it in opposite directions.

Bending: When two complementary forces are acting on a material or object at the same time. (tension, and compression)

Now let's discuss a few examples of these forces.

Imagine you have some marshmallows. 5 large, soft, puffy, sweet marshmallows. Let's imagine we set one of those marshmallows on the table in front of you.

What do you think will happen to that marshmallow if you press down on it with your hand?

Which force do you think is at work?

Let's take another marshmallow, because our last one is no longer useful. Place it on the table in front of you.

What do you think will happen if you pick the marshmallow up, and grasp each end, and gently pull?

Which force do you think is at work here?

Well, we need another marshmallow. That last one is no longer useful either.

Holding the new marshmallow, what do you think will happen if you grasp it on one end, and twist the other end 1 full turn? Can you twist it another full turn?

Which force do you think is at work here?

Wow, 3 marshmallows down. Let's grab the fourth marshmallow.

What do you think will happen to the marshmallow if you use a pair of scissors and cut it?

Which force do you think is at work here?

Only one marshmallow left!

If we hold that marshmallow at each end, and bend it in the middle, what do you think is going to happen?

Which force do you think is at work here?

Now hold that same marshmallow in the middle where you bent it, in one hand, and gently pull down on either end.

What happened? Which force is at work here?

It is amazing we can learn so much about forces that act on structures, from playing with marshmallows!

Let's look at another example of forces on structures, and this time we need 2 uncooked pieces of spaghetti. One whole noodle, and one half noodle. Remember they need to be uncooked!

We will also need two coffee cups, preferably equal heights/ sizes. Turn them upside on your table, and place your long piece of spaghetti on top of the cups, with one end on one cup, and the other end on the other cup. Similar to the example below.

What do you think is going to happen when you take one finger, and begin to gently press down on the middle of the piece of spaghetti?

Which force do you think is at work?

Now let's do the same thing again, but with our shorter piece of spaghetti. Move the cups closer together as needed to support it.

What do you think is going to happen when you press down on the center of this piece of spaghetti?

Is it the same as with the longer piece? What is different?